OBSERVER BASED H_∞ CONTROL FOR UNCERTAIN NONLINEAR NETWORK SYSTEM WITH PACKET DROPOUTS

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DEDICATION

I dedicate this work to my father and mother for their prayer and devotion to me. Also, to my wife who supports me during this time.

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ABSTRACT

Name: Abdullah Saeed Al-Shehri.

Title: Observer Based H_∞ Control for Uncertain Nonlinear Network Systems with Packet Dropouts

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In this work, robust H_∞ control based on state observer and state feedback gains of a class of uncertain nonlinear networking control system (NNCS) has been presented. First, based on some results in the literature, modified results for certain nonlinear networking control system (NNCS) were presented. Then the modified results were extended to include uncertainties. An initial derivation is created to find LMI approach to stabilize a robust control for a second class of NNCS. New LMIs were derived to guarantee robust stability and to calculate the controller gain. The stability of the closed loop system has been tested by a numerical example and compared with exist results in the literature.

ملخص الرسالة

الاسم الكامل:عبدالله بن سعيد الشهري

عنوان الرسالة: التحكم بالإستناد على مرقابة H_∞ للأنظمة الشبكية اللا خطية الغير مؤكدة مع وجود حزم التسريب

التخصص: هندسة النظم قسم اللأت الدقيقة والتحكم

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في هذا العمل تحكم قوي من نوع H_∞ بني على أساس وضعية الرقابة وعلى أساس محصلة وضعية ردة أفعال أصناف محددة من النظم الشبكية اللاخطية الغير مؤكدة (NNCS). في بادية الأمر تم عرض تعديلات على نتائج أوراق عمل سابقة لصنف من النظم الشبيكة اللاخطية، بعد ذلك تم تمديد نتائج هذه التعديلات لتشمل الاوضاع الغير مؤكدة لهذه النظم.
و لقد تم إستنتاج نهج من نوع المتراجحات المصفوفية الخطية (LMI) لتصميم محصلة تقوم بعملية إستقرار في عناصر التحكم القوية لفئة أخرى من النظم الشبيكة الاخطية، تم إشتقاق نهج جديد من نوع المتراجحات المصفوفية الخطية (LMI) لضمان الأستقرار القوي للمتحكمات التي تقوم عليها ولحساب محصلاتها. لقد تم إختبار إستقرار هذه الأنظمة بواسطة مثال رقمي وتم مقارنتها مع نتائج لبحوث مسبقة .

Chapter 1

INTRODUCTION

1.1 NCS Definition

1.1.1 Defining the NCS

Networked Control System (NCS) is a control system that sends its control parameters and the feedback parameters through a real time communication network. It can share these parameters with other nodes outside this system. It is possible that for the controlled systems to be in a remote area from the controller and it is also possible to be controlled by many controllers in the same time. NCSs can be a shared network connecting different networked control systems either in the form of nodes with a single system in each node or with multiple systems in each node. The communication of this type of systems is distributed among the actuators and the sensors.

1.1.2 NCS Types

The control over network can be categorized in two types based on the distribution view. They can either be shared-network control systems or remote control systems. More details regarding the advantages and suitability of each type are explained in [50].

1.2 Features of NCS and Difficulties

NCS tilted toward the distributed control environments through the communication networks. This lead to integrated systems contain the data acquisitions nodes (or calling the data from the sensors), command nodes (or performing the instructions in controllers nodes) and the executers of the instructions nodes (or the actuators and the final control elements). These integrated systems began to be applied in various networks to cover the real-time applications.

1.2.1 Features of NCS and Benefits

The NCS can be used widely through the industry, the medicine and the education applications which lead the researchers to study the possibilities and the abilities to maintain the secrecy of the exchanged information and data. The quality indicators for the controlled plants and the equipment performance become one of the main concerns. Also, the security of the information and the quality of the data accessing are the major concerns in the field of the control systems through the communication networks. As a results the quality of the control and the quality of the service are the main issues for NCS, for more details see papers [22],[23],[24]and [25] . The use of NCS can mean a reduction in the cost of implementation of the wires and the connections points in working systems. Also, the implementation time will be reduced. The possibility to diagnose the failure in the systems will be reduced. The flexibility to make modifications and to transfer the parts becomes higher and can be done in a shortest possible time. In addition to that the cost of the maintenance will be reduced and will be easy. There are more benefits of it but the previous mentioned points are the majors

1.2.2 Difficulties in NCS

The NCS in general face three major difficulties as it is outlined in the following:

1- The limitation of the capacitance in the communication and the constraints in the networks is a major problem.This can happen because of the physical nature of the medium that is used in these networks. The possible effect of this problem is the restrictions in scheduling for data path and it can lead to time variation in the communication. This problem can be treated by the improvement in the structure of the networks, the protocol used in these networks, the data scheduling, and the distribution of the transmitted packets and so on. This subject is not covered in this thesis.

2- The time delay of the data is another major concern in NCS. This issue can happen either from the controller to the actuator or from the sensor to the controller. It can cause delay of completing the data on the suitable time which will lead to wrong decision for the actuation and will affect in the stability of the control. It can cause the packet losses if the data does not reach in appropriate time. There is possibility for the time delay to be merged with the packet loss which leads to general packet dropouts, for more details see [41]. This subject too, has not been given covered in this thesis.

3- The packet dropouts or packet losses mean that there is possibility to lose some data or information either before it reaches to the controller from the sensor or after it reaches the actuator from the controller. In other words, it is attenuation of the data or it can be disordering of data. The researches for this part are shown in chapter two in more details.

The concentration in this thesis is on the packet dropouts. The following sections, after chapter two, dealt with the models that have been created to find a solution for the stability problem. The considered plants in this thesis will be considered after the quantization.

1.3 Stability for NCS with Packet Dropouts

In reference [42] it is demonstrated that there are a lot of research made in the effects of packet dropouts on the NCS. It was found that the probability of losing single packet is the largest. Therefore if the single packet loss is repaired, majority of the problems in the packet dropouts will be solved in the stability of the control. So, using the conventional methods in solving data packet dropouts will be effective. There are some famous methods of finding the solution for the stability in systems with packet loss which can be defined in the following subsection.

1.3.1 Famous Methods for Finding the Stability of NCS

Based on [42], there are famous methodologies that can be used to solve the packet loss in systems but they are not the only methods. These methods can be summarized as follows:

First method is by improving the ways of data packet content and it can be applied for the continuous data dropouts during a predictive period. The content of the data will be taken in the consideration if it comes within a dedicated time. Otherwise, there will be interpolation between the initial driven ways (either time-driven or event-driven) of the reading from the sensor to the controller. This will not be taken as an effective command to the actuator until the data content passes through a defined time for the normal event-driven way.

The second method is the Forward Error Correction (FEC) for real-time repairing packets loss based on an appropriate algorithm. The characteristic for repairing data packet is determined as a result of these algorithms. After that, the data packet loss will be rebuilt.

The third method is the optimal compensator design for data packets loss depending on the measurement for the feedback. It can be designed by considering a model for the measurement which contains a binary random variable for estimating the data packet loss. The measurement model will balance the feedback if there is no dropout or it will estimate the value of the system output. This method will be the base for our thesis for finding the stability of the data packet dropouts in the systems.

Here an observer based for the system will be found to stabilize the system with H_∞ control as an idea for the third famous method. Of course there are other methods for finding the dropout packets but these previous methods are the most famous.

1.3.2 Lipschitz Condition

In this thesis, the nonlinear functions, that have been used, are applicable for the Lipschitz Condition that can be described in the following (it has been taken from sources in Ethernet depending on refer [43])

Definition 1.3.1. *A function satisfies the Lipschitz condition for order δ at $x = 0$ if*

$$|f(h) - f(0)| \leq L\,|h|^{\delta}$$

for all $|h| \leq \alpha$,where $L$and δ are independent from h , $\delta > 0$ and θ is an upper bound for all δ for which a finite L exists.

It has been considered in our models that this condition was applicable in each case in order to simplify the method of finding the suitable LMI approach for each of them.

1.3.3 Lyapnuov's Functions

It is famous method to prove the stability of any system by using the Lyapnuov's function, see [44] and [45].There are many methods that are created on this function and theorems are created based on this function. In this thesis the Laypnuov Like method (LLM) has been our way for proving the stability of our models. Here is a brief description for the conditions and the theorem of this method.

Lyapunov-Like Methods The stability was studied by many mathematicians. To make a proof for stability for the systems, there is a need for creating conditions in the different class of them. In [46], the discrete stochastic systems are defined in the following statements.

Definition 1.3.2. *The origin of a discrete stochastic process x_n is said to be asymptotically stable in mean square if there exist constant $0 < \alpha \leq 1, k_1 \geq 0, k_2 \geq 0$ such that*

$$\mathbb{E}\,\|x_n\|^2 \leq k_1 + k_2(1-\alpha)^n,$$

and then x_n is said to be exponentially bounded in mean square with exponent α.

The definition above does not imply that $\mathrm{E}\|x_n\|^2$ decreases for all $n,$ only the bound decreases exponentially and as $n \to \infty$ the mean square of the process is bounded by $\mathrm{E}\|x_\infty\|^2 \leq k_1$

where k_1 depends on the noise disturbing the system. If $k_1 = 0$ that is, if

E$\|x_n\|^2 \leq k(1-\alpha)^n$,then x_n convergence to zero in mean square (which also implies convergence in probability).

The discrete system can be described by the following equation

$$x_{n+1} = f(x_k) + g(x_k)w_k \tag{1.1}$$

where $x_k^T = (x_k^{1^T}, x_k^{2^T})\ and\ f(x_k)$ are vectors in $R^{2n}, x_k^1\ and\ x_k^2$ are vectors in R^n, A matrix function for the values with dimension $2n \times m$ is represented by$g(x_k)$ and w_k is vector with m dimensional sequence of uncorrelated normalized Gaussian random variables. To apply the LLM on it, this will lead to introduce the following theorem that was stated in [46] as the following

Theorem 1.3.1. *If x_n is generated by the system in 1.1 and if there exists a function $V(\epsilon_n)$ $(with\ \epsilon_n = x_n^1 - x_n^2, \epsilon_0 = a)$ such that*

a) $V(\epsilon_n) \geq c\,\|\epsilon_n\|^2$ *, for all* $\epsilon_n \in R^n, c > 0, V(0) = 0$

b) $\mathbb{E}_{\epsilon_k} V(\epsilon_{n+1}) - V(\epsilon_n) \leq k - \alpha V(\epsilon_n)$*, for all* $\epsilon_n \in R^n, k > 0, 0 < \alpha \leq 1$

Then

$c\mathbb{E}\,\|\epsilon_n\|^2 \leq (1-\alpha)^n V(a) + k\sum_{i=0}^{n-1}(1-\alpha)^i$

If these stochastic process ϵ_n under the previous theorem this will give it a sufficient condition to be asymptotically stable in mean square for the large n. Since as n will go to ∞, E$\|\epsilon_\infty\|^2$ will be less than or equal $\frac{k}{c\alpha}$ regardless the initial state ϵ_0.

So in this thesis the proofs have been built based on deriving the conditions of the system under stochastic dropout that had led to its stability in the mean square sense. Since it was discrete, this has led to find the situation that makes $\Delta V(\epsilon_n) < 0$.

1.3.4 Bernoulli Distribution

The probability for fail and success trail is famous type of events. The general probability function for this type of event is called Binomial distribution and it has two probability either p =success probability or $q = 1 - p =$ the failure probability.If the outcomes of the events is only two then this will be dealt as a specific random distribution which is the Bernoulli distribution .In our models, we will deal the probability of the packet loss in the NCS as two binary events either done or not. As mentioned in [40] if the packet loss is done in one channel will make the majority of the effects that leads us to neglect the the other multi packet losses and the events of it will be considered independent.

The mean and the variance of this distribution will be

$\mu = p \; and \; \sigma^2 = p(1-p)$

1.4 Thesis Outline

The contribution of this thesis was based on the previous principles and it has been ordered in the following five chapters

Chapter 2

It contains the literature survey for the packet dropouts' researches on NCS. It has been divided into two section one for the linear NCS and the second for the nonlinear NCS. It has been written in summarized way without going in the details.

Chapter3

It gives a reproducing for the work of Jian Guo Li and his friends in [4] but by adding the modification of their works. The resulted theorem and its new LMIs are tested with the same example.

Chapter 4

It contains a modeling for the LMIs that have been created in the previous chapter but by adding the uncertainty on it. The test with the numerical example has been made to see the results of the created models

Chapter5

A summary for the previous chapters has been written and comparisons for its results. Also, it contains the future works after making a trail to find a model for another class of nonlinear NCS. The theorem has been created and the test numerical example has been considered as future works based on the free time possibility.

Notations:

$\Pr\{\cdot\}$ means the occurrence probability of the event " $\cdot$ ".$\mathbb{E}\{x\}$ means the expectation of the stochastic variable $x.l_2[0,\infty)$ means the space of square integral vectors. $\mathbb{I}^+$ means the set of the positive integers..$\mathbb{R}$ means the real numbers and $\mathbb{R}^n$ means the n-dimensional of Euclidean space.$\mathbb{R}^{n\times m}$ means the set of all $n\times m$ real matrices.$\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ mean the maximum eigenvalue of matrix A and the minimum eigenvalue of matrix A respectively.$\|\cdot\|$ means Euclidean vector norm or the induced matrix $2-norm.I$ means the identity matrix with appropriate dimension. $X>Y$ or $(X\geqslant Y)$ means $X-Y$ are definite matrix (semi-definite matrix) where X and Y are symmetric matrices. $diag(a_1,a_2,.....a_n)$ means block-diagonal matrix." $*$ " is used as an ellipsis for terms induced by the symmetry in symmetric block matrices.

Chapter 2

LITERATURE REVIEW

In general, NCS researches concentrate in two major issues either research in the control of the network or in the control over network. The study for the control of networks concentrates on the communications and the networks that will be used for the real time NCSs, e.g., routing control, congestion reduction, efficient data communication, networking protocol, etc. The other type is looking for the quality of the control for the plants through the networks to minimize the negative effects of the networks on the NCS performance such as network delay. It is clear from all these researches that the goal of them was to find high in QoS (Quality of the Services) and high QoC (Quality of Control).This thesis is under the QoC. The survey of this thesis was constraining here on the stability of NCS. There is tremendous literature that covers these subjects. To make it easy for the survey it is divided into two major parts: one for the literature review of the linear systems and the other for the nonlinear systems.

2.1 Linear Networked Systems

The stability of the linear systems through NCS was covered hugely to solve the major two problems of the NCS which are the time delay and the packet dropouts. Here a quick review has been made on these researches as shown in the following.

In [20] the author made full study about analysis that how can the NCS be stabilized. His models are linear NCS cases but the major problems that he mentioned can happen in both linear and nonlinear NCS. He categorized the major problems of the NCS into four issues. The first issue is exposing the NCS to the time-varying transmission period because of nondeterministic factors during the transmission process. The second is the network scheduling if there are more than a set of plant control which leads to the possibility of the concurrent transmissions causing a delay or missing of some real-time data. The third is the networked- induced delay that may happen during the data exchange among the devices either constant or varying time delay. The forth issue possibly will happen because of the unreliable paths of the transmissions which the dropout of the data packets and can be missed during the transmission in the network systems. Then he tried to find stability solutions for these four problems but he discussed them in general by using network switch method only.

In [9] the authors assumed that the information is communicated through bandwidth limited network channels that serve multiple users. In order to reduce the complexity, they also assumed that there was no network between the controller outputs and the plant inputs,and there occurred no transmission delay over the communication channel. They considered the stability of the NCSs with communication constraints for two kinds of sparse transmission policies: periodic transmission policy and arbitrary transmission policy. An iterative approach was used such that the controllers can make full use of the previous information to stabilize NCSs when the current state measurements cannot be available from the network.

In [18] the authors used LMI to design a H_∞ control for uncertain linear NCS. They considered the random packet loss.They tried to study the system after the sampling. Their model of the NCS was in the following form.

$$\begin{aligned} x_{k+1} &= (A + \Delta A)x_k + B_2 u_k + B_1 w_k \\ z_k &= (C_1 + \Delta C)x_k + D_1 w_k \end{aligned} \tag{2.1}$$

where $x_k \in \mathbb{R}^n$ is the state vector and it is in linear form, $u_k \in \mathbb{R}^m$ is the control input vector,$z_k \in \mathbb{R}^r$ is the controlled out put vector,$w_k \in \mathbb{R}^q$ is the disturbance input belong to

$l_k[0, \infty)$,and $A, B_1, B_2, C_1 and\ D_1$ are known real matrices with appropriate dimensions. They assumed a random stochastic variable for the communications form the sensor to the controller and another one for the communication from the controller to the actuator. This source was used as a reference for the work in this thesis especially in considering the place of the uncertainty that had been described by the following form

$$\begin{bmatrix} \Delta A \\ \Delta C \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_k E \tag{2.2}$$

where H_1, H_2 and E are known real constant matrices of appropriate dimensions, and F_k is unknown real valued time-varying matrix satisfying $F_k F_k^T \leq I$.

In the behavior of the predictive controller approach to NCSs as shown in [10] and [11], the authors extended the application of the packet-based control approach to the continuous-time case [12]. It has been suggested that it is more difficult to deal with a continuous-time delay than a discrete time when implementing the packet-based control approach to NCSs. This mainly could be due to the fact that the infinite dimension of a continuous time delay system makes the simple extension of the discrete-time case impossible. They dealt with this difficulty, by proposing a concretization technique for the continuous network-induced delay, with which the packet-based control approach developed earlier for discrete-time systems. It can be implemented with minor modifications. By using this packet-based control approach, a novel model for NCSs that can simultaneously deal with network-induced delay, data packet dropout, and data packet disorder is derived. A stability criterion for the closed-loop system was obtained using switched system theory based on average duel time analysis. A stabilized controller design method was also presented to the proposed approach for different network conditions and for different applied feedback gains.

[14] shows that the observer - based approach is one of the major methods in finding the stability. The authors studied a quantized H_∞ control problem for NCSs with both sensor-to- controller packet loss and controller-to-actuator packet loss by introducing an improved random packet-loss model. The quantifier considered there was composed of a dynamic scaling and a static quantifier. Based on this new improved random model, the effect of the packet-loss rate, the upper bound of consecutive packet losses and the quantization on the system performance was studied. An observer-based H_∞ control strategy was proposed to guarantee the closed-loop system exponentially mean-square stable with a prescribed H_∞ disturbance attenuation level.

In [15] the authors considered more general problem of state control around a target state trajectory and not simply the problem of closed-loop system stability. Specifically, it was assumed that a NCS with multiple sensors account for both packet drops and the signal quantization. It was considered that the sensor networks packet losses and signal quantization/encoding were intimately correlated problems. These assumptions led to non-trivial generalization of the classic estimations and the control processes. Also, it led to allow a better characterization of a real NCS. This would be done by assuming a transmission control (TCP)-like protocol [16] between controller and actuator. They solved the problem of an optimum linear quadratic Gaussian (LQG) control [17] around a target state for a stable system. After that, a generalization for unstable systems in case of negligible quantization errors should be provided. They tried to show how the separation principle [17] of the classical control theory did not hold in general because of the optimal estimator that was depending on the used quantifier at controller.

2.2 Nonlinear Networked Systems

The research for the Nonlinear NCS (NNCS) can be considered as extension for the approaches that were mentioned for the linear NCS systems but these methods were applied for the NNCS with the suitable conditions.

Reference [4] is the main reference for this work. The authors intend to deal with the observer-based H_∞ control design problem for a class of networked nonlinear systems with both random sensor-to-controller and controller-to-actuator packet losses. A packet loss there considered to happen because of the communication channels limitation .They considered the packet loss for both ways of communications from the sensor to controller and from the controller to the actuator were different. They modeled the packet loss as two cases either occurred or not occurred by the Bernoulli distribution. The design of the controller and the observer gains were made by the LMI approach. They discussed the stability of the system in the mean square sense.

In reference [19] the authors tried to find the stability of nonlinear systems with packet losses and varying time delay. They used an observer based model to find the stability. Both of the packet loss and the time delay are modeled as Bernoulli distribution. LMI is used to find the controller gain. This reference discussed the stability in the mean square sense.

In [21] the authors tried to find an analysis of robust stability for uncertain nonlinear networked control systems. They found that before 2006 there was a reduction in considering the precision and in studying the weakening points for nonlinear NCS. They noticed that the researchers study deeply on the linear NCS. So, they started the research from modeling the nonlinear systems with induced delay time and considered the uncertainty in the model. Even though they tried to find a stability analysis for the nonlinear NCS with more accuracy but the model was not assumed to face a packet loss and bandwidth limitation.

In [27] the authors provided theoretical results with some assumptions for stabilization of NNCS based on the Model Predictive Control (MPC). The uncertainty was considered there with Network Delay Compensation (NDC) strategy. The constraints suggested there were to help in predicting the model results.

In [28] the authors proposed a robust fault estimator that ensures the fault estimation error is less than prescribed H_∞ performance level, irrespective of the uncertainties and network-induced effects. Their model includes the network-induced delays and packet dropouts in communication channels, which were made by the Markov processes. The bilinear matrix inequalities (BMIs) approach was used with the existence of a delay-dependent fault estimator. An iterative algorithm was proposed to change this non-convex problem into quasi convex optimization problems, which can be solved effectively by available mathematical tools. The fuzzy network model was studied with the uncertainty.

In [30] the authors dealt with Fault Detection (FD) in nonlinear NCSs with induced delay and packet dropout. Observer-based fault detection filter was utilized as residual generator and the fault detection of nonlinear NCSs was formulated as an H_∞ control problem. They tried to detect the fault directly without using the T-S fuzzy-model-based method as it happened in [29]. They tried to achieve simultaneously a nonlinear fault detection filter system model based on appropriate Lyapunov-Krasovskii function, the stochastic stability and the prescribed H_∞ attenuation level. LMIs approach was used with sufficient condition to test for the existence of the desired fault detection. The delay and the packet loss were considered there at the same time.

By observing the previous research, as it mentioned in [35], most available works on NNCS concentrate on the stabilization of equilibrium points, while very few studies address the tracking control of NCS, see [31], [32], [34]. In [35] the authors mentioned that the latter references have shown that the difficulties were not present in the stabilization of equilibrium of NCS only. They found that the controllers are often composed of a feedback term (to ensure the convergence to the desired solution) and a feed forward term (which induces the desired solution in the closed-loop system). By referring to the references [31], [32] and [34] it was shown that the approximate tracking for the errors indicates for induced errors done by the network on the feed forward term. The convergence of the error tracking can be affected by the reference signals which are transmitted via the communication channel. As a consequence, they proposed a method to design controllers which ensure a state tracking objective for NCS affected by exogenous perturbations. Compared to [31], [32], [34], they considered nonlinear systems affected by disturbances (as opposed to linear systems) and they studied the effect of scheduling. They followed an emulation-like approach as in [33], where the design of the controller was done in the absence of communication constraints. After that, they implemented their controller over a network and they tried to find conditions to allow them maintaining the tracking property up to some errors caused by the network. They have tried to ensure the communication between the controller, the plant and the reference systems. The tracking and the connections approximation was done based on running some protocol and by proposing a new protocol to investigate the control situation through the NCS with less errors.

It is proposed in [36] a simultaneously computation for (with only one LMI) the observer and controller gains. The authors reduced the conservatism introduced by the two computing steps. They could have simultaneously different performances requirements on the observer and the controller gains (settling time, disturbance attenuation, for example). Their method was first based on Lyapunov theory with the use of the DMVT (differential mean value theorem) .Then it went toward the solution of the bilinear matrix inequality (BMI) to prove the stability of the systems. After that, they provided new LM conditions which help in the H_∞ stabilization of a class of nonlinear discrete time systems. They utilized some suitable algebraic transformations (Schur Lemma and Young inequality) on the BMI which contains the observer and the controller gains.

Chapter 3

OBSERVER-BASED H_∞ CONTROLLER FOR NCS WITH RANDOM PACKET LOSSES

3.1 Introduction

Here the design of the controller of NNCS will be done on the existing of the packet loss. It was suggested that the plant to be controlled after the sampling and there should be no time delay in the channels. The type of the NNCS controller will be an observer-based. The analysis in this chapter is a reproduction for the work of Jian Guo Li with the others [4] in addition to a new modification of their way of linearization of Bilinear Matrix Inequality (BMI). They designed the controller by the observer based method to achieve the stability of the system using the feasibility of H_∞ optimization through LMI approach and applied it for one class of the NNCS. They have represented the existence of the packet loss by linear function of the stochastic variable satisfying a Bernoulli random binary distribution. They have considered that random variable of the sensor-to-controller packet loss is different than controller-to-actuator packet losses. Finally, a simulation example was presented to demonstrate the effectiveness of the proposed LMI approach. In their method it is required to find matrix equality to see the feasibility of the LMI. Finding the unknown parameters will increase the amount of the coupling between the equations. As consequence, this will increase the numbers of the unknown parameters. Also, in their method it is suggested that definite symmetric matrices of LLM(Lyapunov Like Method) P and Q are the same which is not correct in every case. So in this chapter, a design for NNCS controller has been based on observer-based method through LMI approach. It has been made to find a stable system achieving H_∞ performance in the existence of white noise. It will be done with the new modification that will solve the LMI approach instead of creating matrix equality for them.

3.2 Problem Formulation

The configuration of the NCSs considering packet losses is shown in figure 3.1.The controlled plant is a nonlinear system. The random packet losses occur, simultaneously, in the communication channels both from the sensor to the controller and from the controller to the actuator. Similar to [4], it has been supposed that the data is transmitted in single-packet manner with the same transmission length, and employ the point-to-point network allowable data dropout rate.Also, it has been suggested that the network is point-to-point

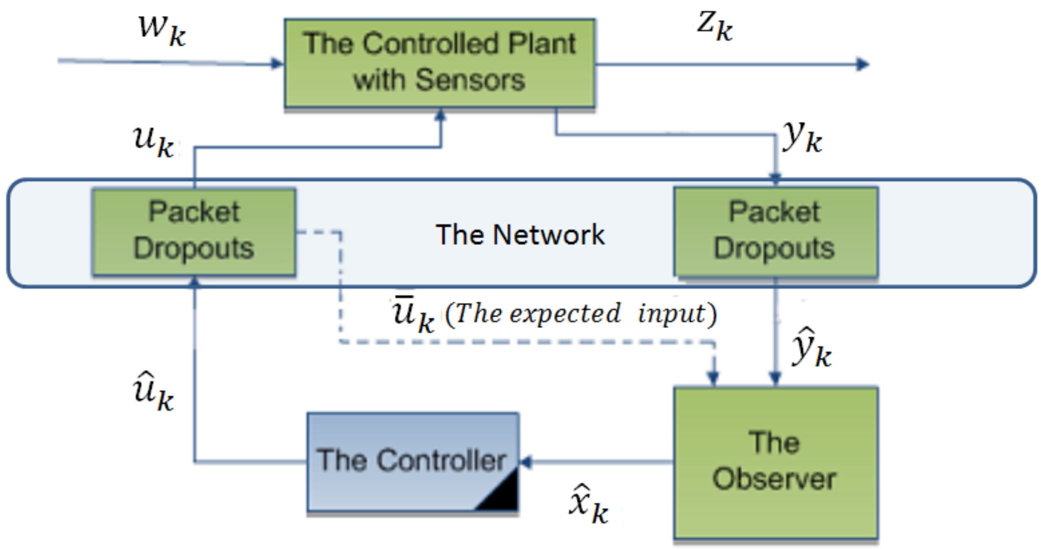

FIGURE 3.1. The layout of the system

throughout to evaluate the QoS of the investigated networked nonlinear system.

In this paper,it has been supposed that a sampled-data model can be obtained through online measurement. Consider the following networked nonlinear control systems after sampling:

$$\begin{aligned} x_{k+1} &= Ax_k + f(k, x_k) + Bu_k + Dw_k \\ z_{k+1} &= C_1x_k + D_1w_k \end{aligned} \tag{3.1}$$

Where $x_k \in \mathbb{R}^n$ is the state vector,$u_k \in \mathbb{R}^m$ is the control input vector,$z_k \in \mathbb{R}^r$ is the controlled output vector,$w_k \in \mathbb{R}^q$ is the disturbance input belong to $l_2[0, \infty)$,$A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, D \in \mathbb{R}^{n \times q}, C_1 \in \mathbb{R}^{r \times n}, D_1 \in \mathbb{R}^{r \times q}$ are known constant matrices.

$f(k, x_k)$ is nonlinear vector function satisfies the global Lipschitz condition:

$$\|f(k,x)\| \leqslant \|Gx\| \tag{3.2}$$

$$\|f(k,x) - f(k,y)\| \leqslant \|G(x-y)\| \tag{3.3}$$

where G is a known real constant matrix.

Remark 3.2.1. *The state matrix in the considered networked control system (3.1) is a nonlinear system while in [5], the networked control system considered there is a linear system.*

The equation of the measurements has been considered to have a random communication packet loss in addtion to the effect of the external noise and it can be described as

$$\hat{y}_k = \alpha_k C_2 x_k + D_2 w_k \tag{3.4}$$

where $\widehat{y}_k \in \mathbb{R}^p$ is the measured output vector,$C_2 \in \mathbb{R}^{p\times n}$, $D_2 \in \mathbb{R}^{p\times q}$ are real constant real matrices.The stochastic variable $\alpha_k \in \mathbb{R}$ is a Bernoulli distributed white sequence with

$$\begin{aligned} \Pr\{\alpha_k &= 1\} = \mathbb{E}\{\alpha_k\} = \bar{\alpha} && (3.5) \\ \Pr\{\alpha_k &= 0\} = 1 - \mathbb{E}\{\alpha_k\} = 1 - \bar{\alpha} && (3.6) \\ Var\{\alpha_k\} &= \mathbb{E}\{(\alpha_k - \bar{\alpha})^2\} = (1 - \bar{\alpha})\bar{\alpha} = \alpha_1^2 && (3.7) \end{aligned}$$

where α_k will represent the random Bernoulli distribution of packet loss from the sensor to the controller and it has been considered as a linear stochastic variable.Similar to [4], the dynamic observer-based control scheme for the networked nonlinear system (3.1) has been described by the observer:

$$\begin{gathered} \hat{x}_{k+1} = A\hat{x}_k + f(k, \hat{x}_k) + B\bar{u}_k + L(\hat{y}_k - \bar{\alpha}C_2\hat{x}_k) \\ \bar{u}_k = \bar{\beta}\hat{u}_k \end{gathered} \qquad (3.8)$$

and by the controller

$$\begin{gathered} \hat{u}_k = -K\hat{x}_k \\ u_k = \beta_k\hat{u}_k \end{gathered} \qquad (3.9)$$

where$\hat{x}_k \in \mathbb{R}^n$ is the state estimate of the networked non linear system (3.1),$\bar{u}_k \in \mathbb{R}^m$ is the control input of the observer,$\hat{u}_k \in \mathbb{R}^m$ is the control input without random packet loss,$u_k \in \mathbb{R}^m$ is the control input of the controlled system,$L \in \mathbb{R}^{n\times p}$ is the observer gain and$K \in \mathbb{R}^{m\times n}$is the controller gain. L and K are the two parameters should be determined.

The stochastic variable $\beta_k \in \mathbb{R}$ is a Bernoulli distributed white sequence with

$$\Pr\{\beta_k = 1\} = \mathbb{E}\{\beta_k\} = \bar{\beta} \tag{3.10}$$

$$\Pr\{\beta_k = 0\} = 1 - \mathbb{E}x\{\beta_k\} = 1 - \bar{\beta} \tag{3.11}$$

$$Var\{\beta_k\} = \mathbb{E}\{(\beta_k - \bar{\beta})^2\} = (1-\bar{\beta})\bar{\beta} = \beta_1^2 \tag{3.12}$$

where β_k represent the random Bernoulli distribution of the packet loss from the controller to the actuator and it has been considered as a linear stochastic variable too.

It has been mentioned in [4] that the random packet-loss model in (3.4) and (3.9) was first introduced in [40] . As shown in figure (3.1),the existence of the random packet loss in the communication channel has been represented as an expected value of it in the observer. The input of the controlled system is different than the input that comes from the controller directly . The difference between them was by adding the stochastic variable factor of the communication loss β_k.in the input of the controlled plant.The feed back of the system can be represented by the error that can be described by

$$e_k = x_k - \hat{x}_k \tag{3.13}$$

A substitution for (3.4) and (3.8)-(3.9) into (3.1) and (3.13) which can be shown in the following steps. First $\hat{u}_k = -K\hat{x}_k$ has been substitute for in $\bar{u}_k = \bar{\beta}\hat{u}_k$ and in $u_k = \beta_k \hat{u}_k$ as shown in the following

$$\begin{aligned} \bar{u}_k &= -\bar{\beta} K \hat{x}_k \\ u_k &= -\beta_k K \hat{x}_k \end{aligned}$$

x_{k+1} can be described by

$$x_{k+1} = Ax_k + f(k, x_k) + Bu_k + Dw_k$$

or it can be described by

$$x_{k+1} = Ax_k + f(k, x_k) - \beta_k BK\hat{x}_k + Dw_k$$

From (3.13) $\hat{x}_k = x_k - e_k$.So

$$
\begin{aligned}
x_{k+1} &= Ax_k + f(k, x_k) - \beta_k BK(x_k - e_k) + Dw_k \\
&= (A - \beta_k BK)x_k + f(k, x_k) + \beta_k BKe_k + Dw_k + \bar{\beta}BKx_k \\
&\quad -\bar{\beta}BKx_k + \bar{\beta}BKe_k - \bar{\beta}BKe_k \\
&= (A - \bar{\beta}BK)x_k + (\beta_k - \bar{\beta})BKe_k - (\beta_k - \bar{\beta})BKx_k \\
&\quad +\bar{\beta}BKe_k + f(k, x_k) + Dw_k
\end{aligned}
$$

Also $\hat{x}_{k+1} = A\hat{x}_k + f(k, \hat{x}_k) + B\bar{u}_k + L(\hat{y}_k - \bar{\alpha}C_2\hat{x}_k)$ and after substituting it became as the following $\hat{x}_{k+1} = (A - \bar{\beta}BK)\hat{x}_k + f(k, \hat{x}_k) + L(\hat{y}_k - \bar{\alpha}C_2\hat{x}_k)$. By considering $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$, the resultant feedback error has been expressed by the following equation

$$
\begin{aligned}
e_{k+1} &= [(A - \bar{\beta}BK)x_k + (\beta_k - \bar{\beta})BKe_k \\
&\quad -(\beta_k - \bar{\beta})BKx_k + \bar{\beta}BKe_k + f(k, \hat{x}_k) + Dw_k] \\
&\quad -[(A - \bar{\beta}BK)\hat{x}_k + f(k, \hat{x}_k) + L(\hat{y}_k - \bar{\alpha}C_2\hat{x}_k)]
\end{aligned}
$$

$$
\begin{aligned}
&= (A-\bar{\beta}BK)x_k + (\beta_k-\bar{\beta})BKe_k \\
&\quad -(\beta_k-\bar{\beta})BKx_k + \bar{\beta}BKe_k + f(k,\hat{x}_k) + Dw_k \\
&\quad -(A-\bar{\beta}BK)x_k + (A-\bar{\beta}BK)e_k) \\
&\quad -f(k,\hat{x}_k) - \alpha_k LC_2x_k - LD_2w_k \\
&\quad +\bar{\alpha}LC_2x_k - \bar{\alpha}LC_2e_k
\end{aligned}
$$

$$
\begin{aligned}
&= -(\beta_k-\bar{\beta})BKx_k - (\alpha_k-\bar{\alpha})LC_2x_k \\
&\quad +(A-\bar{\alpha}LC_2)e_k + (\beta_k-\bar{\beta})BKe_k \\
&\quad +f(k,x_k) - f(k,\hat{x}_k) + (D+LD_2)w_k
\end{aligned}
$$

This has resulted with the following closed-loop networked nonlinear system:

$$
\begin{aligned}
x_{k+1} &= (A-\bar{\beta}BK)x_k + (\beta_k-\bar{\beta})BKe_k - (\beta_k-\bar{\beta})BKx_k \\
&\quad +\bar{\beta}BKe_k + f(k,x_k) + Dw_k \\
e_{k+1} &= -(\beta_k-\bar{\beta})BKx_k - (\alpha_k-\bar{\alpha})LC_2x_k + (A-\bar{\alpha}LC_2)e_k \\
&\quad +(\beta_k-\bar{\beta})BKe_k + F_k + (D+LD_2)w_k
\end{aligned}
\tag{3.14}
$$

where $F_k = f(k,x_k) - f(k,\hat{x}_k)$.Then, by defining

$$
\eta_k = \begin{bmatrix} x_k \\ e_k \end{bmatrix} \tag{3.15}
$$

the closed loop nonlinear system in (3.14) can be described in the following compact form

$$\eta_{k+1} = \bar{A}\eta_k + (\beta_k - \bar{\beta})\hat{A}_1\eta_k + (\alpha_k - \bar{\alpha})\hat{A}_2\eta_k + \bar{F}_k + \bar{B}w_k \tag{3.16}$$

where $\bar{A} = \begin{bmatrix} A + \bar{\beta}BK & \bar{\beta}BK \\ 0 & A - \bar{\alpha}LC_k \end{bmatrix}$, $\hat{A}_1 = \begin{bmatrix} -BK & BK \\ -BK & BK \end{bmatrix}$,
$\hat{A}_2 = \begin{bmatrix} 0 & 0 \\ -LC_2 & 0 \end{bmatrix}$, $\bar{F}_k = \begin{bmatrix} f(k, x_k) \\ F_k \end{bmatrix}$,
$F_k = f(k, x_k) - f(k, \hat{x}_k)$,$\bar{B} = \begin{bmatrix} D \\ D - LD_2 \end{bmatrix}$

This closed loop nonlinear system network, contains the stochastic parameters α_k, β_k. In this chapter, the objective was designing the observer (3.8) and the observer-based controller (3.9) for the networked nonlinear system (3.1), such that, in the presence of the random packet losses, the closed-loop networked nonlinear system (3.16) sufficiently an exponentially main square stable and its H_∞ performance constraint should be also achieved.

3.2.1 The Major Design Requirements

More specifically, the aim was to design a controller such that the closed-loop networked nonlinear system (3.16) satisfies the following two requirements

1-The closed -loop nonlinear networked system (3.14) is sufficiently exponentially main square stable

2-Under the zero-initial condition , for all nonzero white noise w_k,the controlled output z_k should satisfy the condition

$$\sum_{k=0}^{\infty} E \left\| z_k \right\|^2 < \gamma^2 \sum_{k=0}^{\infty} E \left\| w_k \right\|^2 \tag{3.17}$$

where $\gamma > 0$ is a prescribed scalar.

3.3 Main Results

In this section, the main results have been presented. The following definition and lemma were used.

Definition 3.3.1. *The closed loop networked nonlinear system (3.16) is said to be exponentially mean-square stable, when $w_k = 0$ if there exist constant $\phi > 0$ and $\tau \in (0, 1)$such that*

$$\mathbb{E}\{\|\eta_k\|^2\} \leq \phi\tau^k \mathbb{E}\{\|\eta_0\|^2\}, \quad \forall\eta_0 \in \mathbb{R}^n, k \in I^+ \tag{3.18}$$

Lemma 3.3.1. *([41,42])(S-procedure)). Let $T_i \in \mathbb{R}^{n\times n}(i = 0, 1, 2, ..., p)$ be symmetric matrices. The conditions on T_i $(i = 0, 1, 2, ..., p)$,$\varsigma^T T_o \varsigma > 0, \quad \forall\varsigma \neq 0 \; s.t. \varsigma^T T_i \varsigma \geq 0 \;\; (i =$*

$0, 1, 2, ..., p)$ *hold if there exist* $\tau_i \geq 0$ $(i = 0, 1, 2, ..., p)$ *such that* $T_o - \sum_{i=1}^{p} \tau_i T_i > 0$.

In the beginning, it had been required to find the basic model that has satisfied the two major requirements mentioned in (3.17). After that, the modifications on this model has been made to find its LMI approaches.

3.3.1 Finding the Basic Matrix Inequality

In the following theorem, a sufficient condition has been derived such that the closed-loop networked nonlinear system (3.16) is exponentially mean square stable.

Theorem 3.3.1. *Given the communication channel parameters* $0 \leq \bar{\alpha} \leq 1, 0 \leq \bar{\beta} \leq 1$, *the controller gain matrix* K *and the observer gain matrix* L, *the closed-loop networked nonlinear system (3.16) is exponentially mean square stable if there exist positive-definite matrices*P, Q, *and positive real scalars* $\tau_1 > 0, \tau_2 > 0$ *satisfying the linear matrix inequal-*

ity shown in the following.

$$
\begin{bmatrix}
-P+\tau_1 G^T G^T & * & * & * & * & * & * & * & * \\
0 & -Q+\tau_2 G^T G^T & * & * & * & * & * & * & * \\
0 & 0 & -\tau_1 I & * & * & * & * & * & * \\
0 & 0 & 0 & -\tau_2 I & * & * & * & * & * \\
P(A-\bar{\beta}BK) & \bar{\beta}PBK & P & 0 & -P & * & * & * & * \\
0 & Q(A-\bar{\alpha}LC_2) & 0 & Q & 0 & -Q & * & * & * \\
-\beta_1 PBK & \beta_1 PBK & 0 & 0 & 0 & 0 & -P & * & * \\
-\beta_1 QBK & \beta_1 QBK & 0 & 0 & 0 & 0 & 0 & -Q & * \\
\alpha_1 QLC_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q
\end{bmatrix}
< 0 \tag{3.19}
$$

where $\alpha_1 = [(1-\bar{\alpha})\bar{\alpha}]^{1/2}$ *and* $\beta_1 = \left[(1-\bar{\beta})\bar{\beta}\right]^{1/2}$.

Proof. By using a Lyaqunov function

$$V_k = x_k^T P x_k + e_k^T Q e_k, \tag{3.20}$$

where P, Q are positive definite matrices solution to

$\Delta V_k = \mathbb{E}\{V_{k+1}|x_k, x_{k-1}, x_{k-2}, ..., x_0, e_k, e_{k-1}, ..., e_0\} - V_k$.

ΔV_k can be rewritten in the following form

$\Delta V_k = \mathbb{E}\{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k$ or

$$
\begin{aligned}
\Delta V_k = {} & \mathbb{E}\left\{\left[(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})BKe_k\right]^T\right\} \\
& \times P\mathbb{E}\left\{(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})BKe_k\right\} \\
& +\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k+\bar{\beta}BKe_k+f(k,x_k)\right]^T\right\} \\
& \times P\mathbb{E}\left\{(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})BKe_k\right\} \\
& +\mathbb{E}\left\{\left[(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})BKe_k\right]^T\right\} \\
& \times P\mathbb{E}\left\{(\bar{\beta}-\beta_k)BKx_k+\bar{\beta}BKe_k+f(k,x_k)\right\} \\
& +\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k+\bar{\beta}BKe_k+f(k,x_k)\right]^T\right\} \\
& \times P\mathbb{E}\left\{(\bar{\beta}-\beta_k)BKx_k+\bar{\beta}BKe_k+f(k,x_k)\right\} \\
& +\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k+(\bar{\alpha}-\alpha_k)LC_2x_k\right]^T\right\} \\
& \times Q\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k+(\bar{\alpha}-\alpha_k)LC_2x_k\right]\right\} \\
& +\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k+(\bar{\alpha}-\alpha_k)LC_2x_k\right]^T\right\} \\
& \times Q\mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})BKe_k+F_k\right]\right\} \\
& +\mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})BKe_k+F_k\right]^T\right\} \\
& \times Q\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k+(\bar{\alpha}-\alpha_k)LC_2x_k\right]\right\} \\
& +\mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})BKe_k+F_k\right]^T\right\} \\
& \times Q\mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})BKe_k+F_k\right]\right\} \\
& -x_k^TPx_k-e_k^TQe_k^T
\end{aligned}
$$

Noting that $\mathbb{E}\{(\beta_k-\bar{\beta})^2\}=\beta_1^2, \mathbb{E}\{(\alpha_k-\bar{\alpha})^2\}=\alpha_1^2$. Also it is known that $\mathbb{E}\{(\beta_k-\bar{\beta})\}=\mathbb{E}(\beta_k)-\bar{\beta}=\bar{\beta}-\bar{\beta}=0$, and the same thing for $\mathbb{E}\{(\alpha_k-\bar{\alpha})\}=0$,as a properties of the expecting value of the Bernoulli distribution. So, the above equation can be rewritten in the following form .

$$
\begin{aligned}
\Delta V_k = \mathbb{E}&\left\{\left[(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})BKe_k\right]^T\right\} \\
&\times\mathbb{E}\left\{P(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})PBKe_k-(\beta_k-\bar{\beta})PBKx_k\right\} \\
&-\mathbb{E}\left\{\left[(\beta_k-\bar{\beta})BKx_k+\bar{\beta}BKe_k+f(k,x_k)\right]^T\right\} \\
&\times\mathbb{E}\left\{P(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})PBKe_k-(\beta_k-\bar{\beta})PBKx_k\right\} \\
&+\mathbb{E}\left\{\left[(A-\bar{\beta}BK)x_k+(\beta_k-\bar{\beta})BKe_k\right]^T\right\} \\
&\times\mathbb{E}\left\{\bar{\beta}PBKe_k+Pf(k,x_k)\right\} \\
&-\mathbb{E}\left\{\left[(\beta_k-\bar{\beta})BKx_k+\bar{\beta}BKe_k+f(k,x_k)\right]^T\right\} \\
&\times\mathrm{E}\left\{\bar{\beta}PBKe_k+Pf(k,x_k)\right\} \\
&+\mathrm{E}\left\{\left[-(\beta_k-\bar{\beta})BKx_k-(\alpha_k-\bar{\alpha})LC_2x_k\right]^T\right\} \\
&\times\mathrm{E}\left\{-(\beta_k-\bar{\beta})QBKx_k-(\alpha_k-\bar{\alpha})QLC_2x_k\right\} \\
&+\mathrm{E}\left\{\left[-(\beta_k-\bar{\beta})BKx_k-(\alpha_k-\bar{\alpha})LC_2x_k\right]^T\right\} \\
&\times\mathrm{E}\left\{Q(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})QBKe_k+QF_k\right\} \\
&+\mathrm{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})BKe_k+F_k\right]^T\right\}
\end{aligned}
$$

$$\times \mathrm{E}\big\{-(\beta_k-\bar{\beta})QBKx_k-(\alpha_k-\bar{\alpha})QLC_2x_k\big\}$$

$$+\mathrm{E}\Big\{\big[(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})BKe_k+F_k\big]^T\Big\}$$

$$\times \mathrm{E}\big\{Q(A-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})QBKe_k+QF_k\big\}$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

Then, this previous equation will give the following equation by applying the Bernoulli distribution properties.

$$\Delta V_k=((A-\bar{\beta}BK)x_k)^TP(A-\bar{\beta}BK)x_k+\bar{\beta}(BKe_k)^TP(A-\bar{\beta}BK)x_k$$

$$+f(k,x_k)^TP(A-\bar{\beta}BK)x_k$$

$$+((A-\bar{\beta}BK)x_k)^T\bar{\beta}PBKe_k+\bar{\beta}(BKe_k)^T\bar{\beta}PBKe_k$$

$$+f(k,x_k)^T\bar{\beta}PBKe_k+((A-\bar{\beta}BK)x_k)^TPf(k,x_k)$$

$$+\bar{\beta}(BKe_k)^TPf(k,x_k)+f(k,x_k)^TPf(k,x_k)$$

$$+\beta_1^2\left(BKe_k\right)^TPBKe_k-\beta_1^2(BKx_k)^T\,PBKe_k$$

$$-\beta_1^2\left(BKe_k\right)^TPBKx_k+\beta_1^2(BKx_k)^T\,PBKx_k$$

$$+\left(((A-\bar{\alpha}LC_2)e_k)^TQ(A-\bar{\alpha}LC_2)e_k+F_k^TQ(A-\bar{\alpha}LC_2)e_k\right)$$

$$+\left(((A-\bar{\alpha}LC_2)e_k)^TQF_k+F_k^TQF_k\right)$$

$$+\alpha_1^2\left(LC_2x_k\right)^T\left(QLC_2x_k\right)$$

$$+\beta_1^2\left(BKe_k\right)^TQBKe_k-\beta_1^2(BKx_k)^T\,QBKe_k$$

$$-\beta_1^2\left(BKe_k\right)^T QBKx_k + \beta_1^2\left(BKx_k\right)^T QBKx_k$$

$$-x_k^T P x_k - e_k^T Q e_k^T$$

This will end by the following matrix multiplication

$$\Delta V_k = \begin{bmatrix} x_k \\ e_k \\ f(k, x_k) \\ F_k \end{bmatrix}^T \Lambda \begin{bmatrix} x_k \\ e_k \\ f(k, x_k) \\ F_k \end{bmatrix} \tag{3.21}$$
$$\triangleq \xi_k^T \Lambda \xi_k$$

where $\Lambda = \begin{bmatrix} \Phi_{11} & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ P(A - \bar{\beta}BK) & \bar{\beta}PBK & P & * \\ 0 & Q(A - \bar{\alpha}LC_2) & 0 & Q \end{bmatrix}$

$$\Phi_{11} = (A - \bar{\beta}BK)^T P(A - \bar{\beta}BK) + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$

$$+\alpha_1^2 C_2^T L^T QLC_2 - P$$

$$\Phi_{22} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$

$$+(A - \bar{\alpha}LC_2)^T Q(A - \bar{\alpha}LC_2) - Q$$

$$\Phi_{21} = \bar{\beta}K^T B^T P(A - \bar{\beta}BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$$

The two constraints in (3.2) and (3.3) that say

$$f^T(k,x_k)f(k,x_k) = \|f(k,x)\|^2 \leqslant \|Gx\|^2 = x_k^T G^T G x_k \tag{3.22}$$

$$F_k^T F_k = \|F_k\|^2 \leqslant \|Ge_k\|^2 = e_k^T G^T G e_k \tag{3.23}$$

can be rewritten in the following two inequalities $f^T(k,x_k)f(k,x_k) \leqslant x_k^T G^T G x_k$ and $F_k^T F_k \leqslant e_k^T G^T G e_k$. These two inequalities will lead to the following inequalities.The first is $f^T(k,x_k)f(k,x_k)$ $-x_k^T G^T G x_k \leqslant 0$ or

$$\xi_k^T \begin{bmatrix} -G^T G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_1 \xi_k \leq 0 \tag{3.24}$$

The second is $F_k^T F_k - e_k^T G^T G e_k \leqslant 0$ or

$$\xi_k^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -G^T G & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_2 \xi_k \leq 0 \tag{3.25}$$

After that, those two inequalities can be used with the inequality $\Delta V_k = \xi_k^T \Lambda \xi_k < 0$. By using Lemma 3.3.1,this inequality with the constrains (3.24) and (3.25) holds if there exist positive-definite matricesP, Q and nonnegative scalars $\tau_1 > 0, \tau_2 > 0$ such that

$\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 < 0$ Then, as a results for this $\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 =$

$$\begin{bmatrix} \tilde{\Phi}_{11} & * & * & * \\ \tilde{\Phi}_{21} & \tilde{\Phi}_{22} & * & * \\ P(A-\bar{\beta}BK) & \bar{\beta}PBK & P-\tau_1 I & * \\ 0 & Q(A-\bar{\alpha}LC_2) & 0 & Q-\tau_2 I \end{bmatrix} < 0 \text{ where}$$

$$\tilde{\Phi}_{11} = (A-\bar{\beta}BK)^T P(A-\bar{\beta}BK) + \beta_1^2 K^T B^T PBK$$

$$+\beta_1^2 K^T B^T QBK + \alpha_1^2 C_2^T L^T QLC_2 - P + \tau_1 G^T G$$

$$\tilde{\Phi}_{22} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$

$$+(A-\bar{\alpha}LC_2)^T Q(A-\bar{\alpha}LC_2) - Q + \tau_2 G^T G$$

$$\tilde{\Phi}_{21} = \bar{\beta}K^T B^T P(A-\bar{\beta}BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$$

It can be rewritten in this equivalent form of a MI (Matrix Inequality) as the following

$$\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 =$$

$$\begin{bmatrix} -P+\tau_1 G^T G & * & * & * \\ 0 & -Q+\tau_2 G^T G & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$$

$$+\begin{bmatrix} \breve{\Phi}_{11} & * & * & 0 \\ \breve{\Phi}_{21} & \breve{\Phi}_{22} & * & * \\ P(A-\bar{\beta}BK) & \bar{\beta}PBK & P & 0 \\ 0 & Q(A-\bar{\alpha}LC_2) & 0 & Q \end{bmatrix}$$

where

$$\breve{\Phi}_{11} = (A - \bar{\beta}BK)^T P(A - \bar{\beta}BK) + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$
$$+\alpha_1^2 C_2^T L^T QLC_2$$
$$\breve{\Phi}_{21} = \bar{\beta}K^T B^T P(A - \bar{\beta}BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$$
$$\breve{\Phi}_{22} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$
$$+(A - \bar{\alpha}LC_2)^T Q(A - \bar{\alpha}LC_2)$$

The known s-procedure ,which saying that the matrix of the form $G_i + Z^T OO^{-1}OZ$ is equivalent to $\begin{bmatrix} G_i & Z^T O \\ OZ & -O \end{bmatrix}$ where O is symmetric matrix, $G_i$and$Z$ are real matrices, can be used for $\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2$. As a result this $\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2$can be rewritten as

$$\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 =$$
$$\begin{bmatrix} -P + \tau_1 G^T G & * & * & * & * & * & * & * & * \\ 0 & U_{22} & * & * & * & * & * & * & * \\ 0 & 0 & -\tau_1 I & * & * & * & * & * & * \\ 0 & 0 & 0 & -\tau_2 I & * & * & * & * & * \\ P(A - \bar{\beta}BK) & \bar{\beta}PBK & P & 0 & -P & * & * & * & * \\ \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 & -P & * & * & * \\ \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 & 0 & -Q & * & * \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 & 0 & 0 & -Q & * \\ 0 & U_{92} & 0 & Q & 0 & 0 & 0 & 0 & -Q \end{bmatrix} < 0 \tag{3.26}$$

where $U_{22} = -Q + \tau_2 G^T G$,$U_{92} = Q(A - \bar{\alpha} LC_2)$. After a rearrangement, it will give the same MI that is shown in (3.19).

$$\begin{bmatrix} -P+\tau_1 G^T G & * & * & * & * & * & * & * & * \\ 0 & -Q+\tau_2 G^T G & * & * & * & * & * & * & * \\ 0 & 0 & -\tau_1 I & * & * & * & * & * & * \\ 0 & 0 & 0 & -\tau_2 I & * & * & * & * & * \\ P(A-\bar{\beta}PBK) & \bar{\beta}PBK & P & 0 & -P & * & * & * & * \\ 0 & Q(A-\bar{\alpha}LC_2) & 0 & Q & 0 & -Q & * & * & * \\ -\beta_1 PBK & \beta_1 PBK & 0 & 0 & 0 & 0 & -P & * & * \\ -\beta_1 QBK & \beta_1 QBK & 0 & 0 & 0 & 0 & 0 & -Q & * \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix} <$$

0

Thus we have

$\Delta V_k = \xi_k^T \Lambda \xi_k < 0$ if $\Lambda < 0$ and

$\Delta V_k = \xi_k^T \Lambda \xi_k \leq -\lambda_{\min}(-\Lambda)\xi_k^T \xi_k$ or

$\Delta V_k \leq -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + f(k, x_k)^T f(k, x_k) + F_k^T F_k)$, then

$\Delta V_k \leq -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + \|f(k, x_k)\|^2 + \|F_k\|^2) < -\alpha \eta_k^T \eta_k$

where

$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \sigma\}$

$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \max\{\lambda_{\max}(P), \lambda_{\max}(Q)\}\}$

This will lead to the following

$$\Delta V_k < -\alpha \eta_k^T \eta_k < -\frac{\alpha}{\sigma} V_k := -\psi V_k \tag{3.27}$$

Therefore by the definition (3.18) , it is verified from the Theorem 3.3.1 that the closed-loop nonlinear networked system (3.14) is exponentially mean-square stable . $\square$

Next, the derivation is continued in the sufficient conditions such that not only the closed-loop networked nonlinear system (3.16) is exponentially mean square stable, but also it can achieve the H_∞ performance constraint (3.17). In this paper, (3.17) has been used to describe the H_∞ performance of the stochastic nonlinear system (3.16), where the expectation operator is utilized on both the controlled output and the disturbance input.

Theorem 3.3.2. *Given communication channel parameters* $0 \le \bar{\alpha} \le 1 and\ 0 \le \bar{\beta} \le 1$, *and a scalar* $\gamma > 0$. *The closed loop networked nonlinear system (3.16) is exponentially mean suqare stable and the* H_∞ *performance constraint (3.17) is achieved for all nonzero* w_k, *if there exist positive definite matrices* $P > 0,\ Q > 0$, *real matrices* K, *and* L; *and non negative real scalar* $\tau_1 > 0, \tau_2 > 0$ *satisfying the matrix inequality shown in (3.28a)*

$$\begin{bmatrix} \Phi_{1\,1} & * & * & * & * & * & * & * & * & * & * \\ 0 & \Phi_{2\,2} & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & -\tau_1 I & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\tau_2 I & * & * & * & * & * & * \\ \Phi_{6\,1} & \bar{\beta} PBK & PD & P & 0 & -P & * & * & * & * & * \\ 0 & \Phi_{7\,2} & \Phi_{7\,3} & 0 & Q & 0 & -Q & * & * & * & * \\ C_1 & 0 & D_1 & 0 & 0 & 0 & 0 & -I & * & * & * \\ -\beta_1 PBK & \beta_1 PBK & 0 & 0 & 0 & 0 & 0 & 0 & -P & * & * \\ -\beta_1 QBK & \beta_1 QBK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q & * \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix} < 0 \tag{3.28a}$$

where $\Phi_{1\,1} = -P + \tau_1 G^T G$,$\Phi_{2\,2} = -Q + \tau_2 G^T G$,$\Phi_{6\,1} = P(A - \bar{\beta} BK)$,$\Phi_{7\,2} = QA - \bar{\alpha} QLC_2$,$\Phi_{7\,3} = QD - QLD_2$,$\alpha_1 = [(1-\bar{\alpha})\bar{\alpha}]^{1/2}$ *and* $\beta_1 = \left[(1-\bar{\beta})\bar{\beta}\right]^{1/2}$.

Proof. The non zero w_k and the output z_k can be inserted in the following equation.

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} =$$

$$\mathrm{E}\{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k$$

$$+ \left[C_1 x_k + D_1 w_k\right]^T \left[C_1 x_k + D_1 w_k\right] - \gamma^2 \mathbb{E}\{w_k^T w_k\}$$

The previous equation can be rewritten in the following form.

$$
\begin{aligned}
&\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = \\
&\qquad \mathbb{E}\left\{\left[(A-\bar{\beta}BK)x_k + (\beta_k - \bar{\beta})BKe_k\right]^T\right\} \\
&\qquad \times P\mathbb{E}\left\{(A-\bar{\beta}BK)x_k + (\beta_k - \bar{\beta})BKe_k\right\} \\
&\qquad + \mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k + \bar{\beta}BKe_k + f(k,x_k)\right]^T\right\} \\
&\qquad \times P\mathbb{E}\left\{(A-\bar{\beta}BK)x_k + (\beta_k - \bar{\beta})BKe_k\right\} \\
&\qquad + \mathbb{E}\left\{\left[(A-\bar{\beta}BK)x_k + (\beta_k - \bar{\beta})BKe_k\right]^T\right\} \\
&\qquad \times P\mathbb{E}\left\{(\bar{\beta}-\beta_k)BKx_k + \bar{\beta}BKe_k + f(k,x_k)\right\} \\
&\qquad + \mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k + \bar{\beta}BKe_k + f(k,x_k)\right]^T\right\} \\
&\qquad \times P\mathbb{E}\left\{(\bar{\beta}-\beta_k)BKx_k + \bar{\beta}BKe_k + f(k,x_k)\right\} \\
&\qquad + \mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k + (\bar{\alpha}-\alpha_k)LC_2x_k\right]^T\right\} \\
&\qquad \times Q\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k + (\bar{\alpha}-\alpha_k)LC_2x_k\right]\right\} \\
&\qquad + \mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k + (\bar{\alpha}-\alpha_k)LC_2x_k\right]^T\right\} \\
&\qquad \times Q\mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})BKe_k + F_k\right]\right\} \\
&\qquad + \mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})BKe_k + F_k\right]^T\right\} \\
&\qquad \times Q\mathbb{E}\left\{\left[(\bar{\beta}-\beta_k)BKx_k + (\bar{\alpha}-\alpha_k)LC_2x_k\right]\right\} \\
&\qquad + \mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})BKe_k + F_k\right]^T\right\} \\
&\qquad \times Q\mathbb{E}\left\{\left[(A-\bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})BKe_k + F_k\right]\right\}
\end{aligned}
$$

$$-x_k^T P x_k - e_k^T Q e_k^T$$

$$+\mathbb{E}\left\{[C_1 x_k + D_1 w_k]^T\right\}$$

$$\times\mathbb{E}\left\{[C_1 x_k + D_1 w_k]\right\}$$

$$-\gamma^2 w_k^T w_k \triangleq$$

As it has been mentioned previously to note that $\mathbb{E}\{(\beta_k - \bar{\beta})\} = \mathbb{E}(\beta_k) - \bar{\beta} = \bar{\beta} - \bar{\beta} = 0$, and the same thing for $\mathbb{E}\{(\alpha_k - \bar{\alpha})\} = 0$. Also, the expectation for $\mathbb{E}\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta})\}$can be in the following form $\mathbb{E}\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta})\} = \mathbb{E}\{(\alpha_k\beta_k - \bar{\alpha}\beta_k - \alpha_k\bar{\beta} + \bar{\alpha}\bar{\beta})\} = \mathbb{E}\{(\alpha_k\beta_k)\} - \mathbb{E}\{\bar{\alpha}\beta_k\} - \mathbb{E}\{\alpha_k\bar{\beta}\} + \mathbb{E}\{(\bar{\alpha}\bar{\beta})\}$. As a results for the property of the Bernoulli distribution and the independence property of theses two different stochastic variables,$\mathbb{E}\{(\alpha_k\beta_k)\} - \mathbb{E}\{\bar{\alpha}\beta_k\} - \mathbb{E}\{\alpha_k\bar{\beta}\} + \mathbb{E}\{(\bar{\alpha}\bar{\beta})\}$ will be end with the following result $\mathbb{E}\{(\alpha_k\beta_k)\} - \mathbb{E}\{\bar{\alpha}\beta_k\} - \mathbb{E}\{\alpha_k\bar{\beta}\} + \mathbb{E}\{(\bar{\alpha}\bar{\beta})\} = \bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta} + \bar{\alpha}\bar{\beta} = 0$. So,$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2\mathbb{E}\{w_k^T w_k\}$ can be rewritten in the following form

$$\begin{aligned}
&\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2\mathbb{E}\{w_k^T w_k\} = \\
&\qquad\left[\begin{array}{c} \left((A - \bar{\beta}BK)x_k + \bar{\beta}BKe_k + f(k, x_k) + Dw_k\right)^T \\ \times P\ \left((A - \bar{\beta}BK)x_k + \bar{\beta}BKe_k + f(k, x_k) + Dw_k\right) \end{array}\right] \\
&\qquad + \left[\beta_1^2 \left(BKe_k - BKx_k\right)^T P \left(BKe_k - BKx_k\right)\right] \\
&\qquad + \left[\begin{array}{c} \left((A - \bar{\alpha}LC_2)e_k + F_k + (D - LD_2)w_k\right)^T \\ Q\left((A - \bar{\alpha}LC_2)e_k + F_k + (D - LD_2)w_k\right) \end{array}\right] \\
&\qquad + \alpha_1^2 \left(LC_2 x_k\right)^T Q \left(LC_2 x_k\right)
\end{aligned}$$

$$+\left[\beta_1^2\left(BKe_k - BKx_k\right)^T Q\left(BKe_k - BKx_k\right)\right]$$

$$-x_k^T P x_k - e_k^T Q e_k^T$$

$$+[C_1 x_k + D_1 w_k]^T [C_1 x_k + D_1 w_k] - \gamma^2 w_k^T w_k^T$$

After the multiplication, it can be rewritten in the following form

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 +$$

$$L_8 + L_9 + L_{10} + L_{11} + \alpha_1^2\left(LC_2 x_k\right)^T\left(QLC_2 x_k\right) - x_k^T P x_k - e_k^T Q e_k^T - \gamma^2 w_k^T w_k^T$$

where

$$L_1 = ((A - \bar{\beta}BK)x_k)^T P(A - \bar{\beta}BK)x_k + \bar{\beta}(BKe_k)^T P(A - \bar{\beta}BK)x_k$$

$$+f(k, x_k)^T P(A - \bar{\beta}BK)x_k + (Dw_k)^T P(A - \bar{\beta}BK)x_k$$

$$L_2 = ((A - \bar{\beta}BK)x_k)^T \bar{\beta}PBKe_k + \bar{\beta}(BKe_k)^T \bar{\beta}PBKe_k$$

$$+f(k, x_k)^T \bar{\beta}PBKe_k + (Dw_k)^T \bar{\beta}PBKe_k$$

$$L_3 = ((A - \bar{\beta}BK)x_k)^T Pf(k, x_k) + \bar{\beta}(BKe_k)^T Pf(k, x_k)$$

$$+f(k, x_k)^T Pf(k, x_k) + (Dw_k)^T Pf(k, x_k)$$

$$L_4 = ((A - \bar{\beta}BK)x_k)^T PDw_k + \bar{\beta}(BKe_k)^T PDw_k$$

$$+f(k, x_k)^T PDw_k + (Dw_k)^T PDw_k$$

$$L_5 = \beta_1^2\left(BKe_k\right)^T PBKe_k - \beta_1^2\left(BKx_k\right)^T PBKe_k$$

$$-\beta_1^2\left(BKe_k\right)^T PBKx_k + \beta_1^2\left(BKx_k\right)^T PBKx_k$$

$$L_6 = ((A - \bar{\alpha}LC_2)e_k)^T Q(A - \bar{\alpha}LC_2)e_k + F_k^T Q(A - \bar{\alpha}LC_2)e_k$$

$$+((D - LD_2)w_k)^T Q(A - \bar{\alpha}LC_2)e_k$$

$$L_7 = ((A - \bar{\alpha}LC_2)e_k)^T QF_k + F_k^T QF_k$$

$$+((D - LD_2)w_k)^T QF_k$$

$$L_8 = ((A - \bar{\alpha}LC_2)e_k)^T Q(D - LD_2)w_k + F_k^T Q(D - LD_2)w_k$$

$$+((D - LD_2)w_k)^T Q(D - LD_2)w_k$$

$$L_9 = \beta_1^2 \left(BKe_k\right)^T QBKe_k - \beta_1^2 \left(BKx_k\right)^T QBKe_k$$

$$-\beta_1^2 \left(BKe_k\right)^T QBKx_k + \beta_1^2 \left(BKx_k\right)^T QBKx_k$$

$$L_{10} = (C_1x_k)^T C_1x_k + (C_1x_k)^T D_1w_k$$

$$L_{11} = (D_1w_k)^T C_1x_k + (D_1w_k)^T D_1w_k$$

This has resulted in the following matrix.

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} =$$
$$\begin{bmatrix} x_k \\ e_k \\ w_k \\ f(k, x_k) \\ F_k \end{bmatrix}^T \begin{bmatrix} \Psi_{11} & * & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * & * \\ \Psi_{41} & \bar{\beta}PBK & PD & P & * \\ 0 & \Psi_{52} & Q(D - LD_2) & 0 & Q \end{bmatrix} \begin{bmatrix} x_k \\ e_k \\ w_k \\ f(k, x_k) \\ F_k \end{bmatrix} \triangleq \zeta_k^T \Omega \zeta_k \tag{3.29}$$

where

$\Psi_{11} = (A-\bar{\beta}BK)^T P(A-\bar{\beta}BK)+\beta_1^2 K^T B^T PBK+\beta_1^2 K^T B^T QBK+\alpha_1^2 C_2^T L^T QLC_2+$
$C_1^T C_1^T - P$,

$\Psi_{22} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK + (A - \bar{\alpha}LC_2)^T Q(A - \bar{\alpha}LC_2) - Q$,

$\Psi_{21} = \bar{\beta}K^T B^T P(A - \bar{\beta}BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$,

$\Psi_{33} = D^T PD + (D - LD_2)^T Q(D - LD_2) + D^T D_1 - \gamma^2 I$,

$\Psi_{32} = \bar{\beta}D^T PBK + (D - LD_2)^T Q(A - \bar{\alpha}LC_2)$,

$\Psi_{31} = D^T P(A - \bar{\beta}BK) + D_1^T C_1$,

$\Psi_{41} = PA - \bar{\beta}PBK \ \Psi_{52} = Q(A - \bar{\alpha}LC_2)$,

Also in the same manner, By referring to the two constraints (3.2) and (3.3), the following two MIs have been obtained. The first MI is

$$\zeta_k^T \begin{bmatrix} -G^T G & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Omega_1 \zeta_k \leq 0.$$

It has come from the relation $f^T(k, x_k) f(k, x_k) \leqslant x_k^T G^T G x_k$ *or* $f^T(k, x_k)\ f(k, x_k) - x_k^T G^T G x_k \leqslant 0$. *This second MI is*

$$\zeta_k^T \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -G^T G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Omega_2 \zeta_k \leq 0.$$

It has come from. $F_k^T F_k \leqslant e_k^T G^T G e_k$ *or* $F_k^T F_k - e_k^T G^T G e_k \leqslant 0$.

By the S-procedure $\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = \zeta_k^T \Omega \zeta_k < 0$ *with the constrains (3.24) and (3.25) holds if there exist positive-definite matrices*P, Q *and nonnegative scalars* $\tau_1 > 0, \tau_2 > 0$ *such that*

$$\Omega - \tau_1 \Omega_1 - \tau_2 \Omega_2 < 0 \tag{3.30}$$

This has resulted in the following form

$$\Omega - \tau_1 \Omega_1 - \tau_2 \Omega_2 = \begin{bmatrix} \tilde{\Psi}_{11} & * & * & * & * \\ \tilde{\Psi}_{21} & \tilde{\Psi}_{22} & * & * & * \\ \tilde{\Psi}_{31} & \tilde{\Psi}_{32} & \tilde{\Psi}_{33} & * & * \\ \tilde{\Psi}_{41} & \bar{\beta} PBK & PD & P + \tau_1 I & * \\ 0 & \tilde{\Psi}_{52} & Q(D - LD_2) & 0 & Q + \tau_2 I \end{bmatrix} < 0 \quad \text{(3.31a)}$$

where

$$\tilde{\Psi}_{11} = (A - \bar{\beta} BK)^T P (A - \bar{\beta} BK) + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$

$$+\alpha_1^2 C_2^T L^T Q L C_2 + C_1^T C_1 - P + \tau_1 G^T G$$

$$\tilde{\Psi}_{22} = \bar{\beta}^2 K^T B^T P B K + \beta_1^2 K^T B^T P B K + \beta_1^2 K^T B^T Q B K$$

$$+(A - \bar{\alpha} L C_2)^T Q (A - \bar{\alpha} L C_2) - Q + \tau_2 G^T G$$

$$\tilde{\Psi}_{33} = D^T P D + (D - L D_2)^T Q (D - L D_2) + D^T D_1 - \gamma^2 I$$

$$\tilde{\Psi}_{21} = \bar{\beta} K^T B^T P (A - \bar{\beta} B K) - \beta_1^2 K^T B^T P B K - \beta_1^2 K^T B^T Q B K$$

$$\tilde{\Psi}_{31} = D^T P (A - \bar{\beta} B K) + D_1^T C_1$$

$$\tilde{\Psi}_{32} = \bar{\beta} D^T P B K + (D - L D_2)^T Q (A - \bar{\alpha} L C_2)$$

$$\tilde{\Psi}_{41} = P A - \bar{\beta} P B K$$

$$\tilde{\Psi}_{52} = Q (A - \bar{\alpha} L C_2)$$

3.31a can be written in the following equivalent MI form

$$\begin{bmatrix} -P + \tau_1 G^T G & * & * & * & 0 \\ 0 & -Q + \tau_2 G^T G & * & * & 0 \\ 0 & 0 & -\gamma^2 I & * & 0 \\ 0 & 0 & 0 & -\tau_1 I & 0 \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$$

$$+ \begin{bmatrix} \Theta_{1\,1} & * & * & * & * \\ \Theta_{2\,1} & \Theta_{2\,2} & * & * & * \\ \Theta_{3\,1} & \Theta_{3\,2} & \Theta_{3\,3} & * & * \\ PA - \bar{\beta} PBK & \bar{\beta} PBK & PD & P & * \\ 0 & Q(A - \bar{\alpha} L C_2) & Q(D - L D_2) & 0 & Q \end{bmatrix} < 0$$

where

$$\Theta_{1\,1} = (A - \bar{\beta}BK)^T P(A - \bar{\beta}BK) + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$

$$+\alpha_1^2 C_2^T L^T QLC_2 + C_1^T C_1$$

$$\Theta_{2\,2} = \bar{\beta}^2 K^T B^T PBK + \beta_1^2 K^T B^T PBK + \beta_1^2 K^T B^T QBK$$

$$+(A - \bar{\alpha}LC_2)^T Q(A - \bar{\alpha}LC_2)$$

$$\Theta_{2\,1} = \bar{\beta}K^T B^T P(A - \bar{\beta}BK) - \beta_1^2 K^T B^T PBK - \beta_1^2 K^T B^T QBK$$

$$\Theta_{3\,1} = D^T P(A - \bar{\beta}BK) + D_1^T C_1$$

$$\Theta_{3\,2} = \bar{\beta}D^T PBK + (D - LD_2)^T Q(A - \bar{\alpha}LC_2)$$

$$\Theta_{3\,3} = D^T PD + (D - LD_2)^T Q(D - LD_2) + D^T D_1$$

By using the s-procedure the previouse matrix can be written in the following form.

$$= \begin{bmatrix} \Pi_{11} & \Pi_{21}^T \\ \Pi_{21} & \Pi_{22} \end{bmatrix} < 0 \tag{3.32}$$

where

$$\Pi_{11} = \begin{bmatrix} -P + \tau_1 G^T G & * & * \\ 0 & -Q + \tau_2 G^T G & * \\ 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} -\tau_1 I & * & * & * & * & * & * & * \\ 0 & -\tau_2 I & * & * & * & * & * & * \\ P & 0 & -P & * & * & * & * & * \\ 0 & 0 & 0 & -P & * & * & * & * \\ 0 & 0 & 0 & 0 & -Q & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q & * & * \\ 0 & Q & 0 & 0 & 0 & 0 & -Q & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$\Pi_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ P(A-\bar{\beta}BK) & \bar{\beta}PBK & PD \\ \beta_1 PBK & -\beta_1 PBK & 0 \\ \beta_1 QBK & -\beta_1 QBK & 0 \\ \alpha_1 QLC_2 & 0 & 0 \\ 0 & Q(A-\bar{\alpha}LC_2) & Q(D-LD_2) \\ C_1 & 0 & D_1 \end{bmatrix}$$

After a rearrangement, it gives the same MI shown in (3.28a).Also, the congruence transformation can be made with the followin matrix

$daig[I, I, I, I, I, P^{-1}, Q^{-1}, I, P^{-1}, Q^{-1}, Q^{-1}]$. *This will give the following resultant* *matrix*

$$\begin{bmatrix} U_{11} & * & * & * & * & * & * & * & * & * & * \\ 0 & U_{22} & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & -\tau_1 I & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\tau_2 I & * & * & * & * & * & * \\ U_{61} & \bar{\beta} BK & D & I & 0 & -P^{-1} & * & * & * & * & * \\ 0 & U_{72} & U_{73} & 0 & I & 0 & -Q^{-1} & * & * & * & * \\ C_1 & 0 & D_1 & 0 & 0 & 0 & 0 & -I & * & * & * \\ -\beta_1 BK & \beta_1 BK & 0 & 0 & 0 & 0 & 0 & 0 & -P^{-1} & * & * \\ -\beta_1 BK & \beta_1 BK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q^{-1} & * \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Q^{-1} \end{bmatrix} < 0 \tag{3.33}$$

where

$U_{11} = -P + \tau_1 G^T G$

$U_{61} = (A - \bar{\beta} BK),$

$U_{72} = A - \bar{\alpha} LC_2,$

$U_{73} = D - LD_2$

$U_{22} = -Q + \tau_2 G^T G.$

So by schur complement, the inequilty in (3.28a) is equivalent to (3.30) and the inequilty in (3.33) is equivalent to (3.30) but with one unknown in each element.Thus this ends with *the following inequality*

$$\zeta_k^T \Omega \zeta_k^T < 0 \tag{3.34}$$

It can be concluded that from (3.29) and (3.34) that

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} < 0 \tag{3.35}$$

Now, summing (3.35) from 0 *to* ∞ *with respect to* k *yield*

$$\sum_{k=0}^{\infty} \mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{w_k^T w_k\} - \mathbb{E}\{V_0\} - \mathbb{E}\{V_\infty\} \tag{3.36}$$

That's where , the closed-loop network nonlinear system (3.16) is exponentially mean square stable and $\eta_0 = 0$*, it is straightforward to conclude that*

$$\sum_{k=0}^{\infty} \mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{w_k^T w_k\} \tag{3.37}$$

Which mean the specified H_∞ *performance constraints (3.17) is achieved. This will end the proof.* □

Here, the basic model has been found. The next step was the finding of the suitable approach for this model in order to design its suitable gains.

3.3.2 Finding the Suitable Approach

The previous model needs the use of a very high level and a special logic solver because of its complexity. So, to find the suitable controller gain and the observer gain in a linear environment logic solver, it is needed to find an LMI approach with less constraints. In this thesis, these approaches have been created by adding the following two modifications in the system shown in (3.33). These two modifications are near to each other. The first approach has been found by suggesting the following step. Based on the konwn s-procedure, the matrix in (3.33) can be hold if there is $\Omega_3 > 0$ such that

$$\hat{\Omega} - \Omega_3 < 0$$

where $\Omega_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N_2 \end{bmatrix}$

and $\hat{\Omega}$ is the MI $\Omega - \tau_1\Omega_1 - \tau_2\Omega_2$that has been found in (3.33), N_1, N_2 are matrices with the same dimensions of P, Q respectively. The matrcies N_1, N_2 are found by the following two eqautions $N_1 = (\tilde{N}_1 - P^{-1})$ and $N_2 = (\tilde{N}_2 - Q^{-1})$ where $\tilde{N}_1$and $\tilde{N}_2$ are positive-definite matrices.Therefore

$$\hat{\Omega} - \Omega_3 = \begin{bmatrix} U_{11} & * & * & * & * & * & * & * & * & * & * \\ 0 & U_{22} & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & -\tau_1 I & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\tau_2 I & * & * & * & * & * & * \\ U_{61} & \bar{\beta} BK & D & I & 0 & U_p & * & * & * & * & * \\ 0 & U_{72} & U_{73} & 0 & I & 0 & U_q & * & * & * & * \\ C_1 & 0 & D_1 & 0 & 0 & 0 & 0 & -I & * & * & * \\ -\beta_1 BK & \beta_1 BK & 0 & 0 & 0 & 0 & 0 & 0 & U_p & * & * \\ -\beta_1 BK & \beta_1 BK & 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_q & * \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & U_q \end{bmatrix}$$

$$-\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&N_1&0&0&0&0&0\\ 0&0&0&0&0&0&N_2&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&N_1&0&0\\ 0&0&0&0&0&0&0&0&0&N_2&0\\ 0&0&0&0&0&0&0&0&0&0&N_2 \end{bmatrix} < 0$$

where

$U_{11} = -P + \tau_1 G^T G$

$U_{61} = (A - \bar{\beta} BK),$

$U_{72} = A - \bar{\alpha} LC_2,$

$U_{73} = D - LD_2$

$U_{22} = -Q + \tau_2 G^T G$

$U_p = -P^{-1}$

$U_q = -Q^{-1}.$

It can be rewritten in the following form

$$\begin{bmatrix} \Xi_{11} & \Xi_{12}^T \\ \Xi_{12} & \Xi_{22} \end{bmatrix} < 0 \tag{3.38}$$

where

$$\Xi_{11} = \begin{bmatrix} -P + \tau_1 G^T G & * & * \\ 0 & -Q + \tau_2 G^T G & * \\ 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Xi_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (A - \bar{\beta} BK) & \bar{\beta} BK & D \\ 0 & A - \bar{\alpha} LC_2 & D - LD_2 \\ C_1 & 0 & D_1 \\ -\beta_1 BK & \beta_1 BK & 0 \\ -\beta_1 BK & \beta_1 BK & 0 \\ \alpha_1 LC_2 & 0 & 0 \end{bmatrix},$$

$$\Xi_{22} = \begin{bmatrix} -\tau_1 I & * & * & * & * & * & * & * \\ 0 & -\tau_2 I & * & * & * & * & * & * \\ 0 & 0 & -\tilde{N}_1 & * & * & * & * & * \\ 0 & 0 & 0 & -\tilde{N}_2 & * & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -\tilde{N}_1 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{N}_2 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{N}_2 \end{bmatrix}.$$

This format is needed to derive the explicit expression of the controller parameter in terms of the LMI solver, such as Matlab Toolbox. It has been noted in this format that there were unknown parameters matrices more than the symmetric matrices P and Q which are $\tilde{N}_1$and $\tilde{N}_2$.

Even though this previous model can be solved by the LMI solver , it has been suggested to reduce the number of the unknown parameter matrices in it. The second approach has been created based on the source [42].It can be created from the following two inequalities the relation between $-P^{-1}$ and $P - 2I.$,and between $-Q^{-1}$ and $Q - 2I.$

$(P^{-1} - I)(P^{-1} - I) \geq 0$ and

$(Q^{-1} - I)(Q^{-1} - I) \geq 0$

These previouse inequalities can be in the following two forms

$P^{-2} - 2P^{-1} + I \geq 0$ and

$Q^{-2} - 2Q^{-1} + I \geq 0$

By multiplying both of them with P and Q respectively this will end by the following two forms

$-P^{-1} \leq P - 2I$ and $-Q^{-1} \leq Q - 2I$

So, they can be subtituted in the MI that is mentioned in (3.33). This has resulted with the following LMI form

$$\begin{bmatrix} \Pi_{11} & \Pi_{12}^T \\ \Pi_{12} & \Pi_{22} \end{bmatrix} < 0 \tag{3.39}$$

.where

$$\Pi_{11} = \begin{bmatrix} -P + \tau_1 G^T G & * & * \\ 0 & -Q + \tau_2 G^T G & * \\ 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Pi_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (A - \bar{\beta} BK) & \bar{\beta} BK & D \\ 0 & A - \bar{\alpha} LC_2 & D - LD_2 \\ C_1 & 0 & D_1 \\ -\beta_1 BK & \beta_1 BK & 0 \\ -\beta_1 BK & \beta_1 BK & 0 \\ \alpha_1 LC_2 & 0 & 0 \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} -\tau_1 I & * & * & * & * & * & * & * \\ 0 & -\tau_2 I & * & * & * & * & * & * \\ 0 & 0 & P - 2I & * & * & * & * & * \\ 0 & 0 & 0 & Q - 2I & * & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & P - 2I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & Q - 2I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q - 2I \end{bmatrix}.$$

To establish the observer-based controller (3.8) and (3.9) for the networked nonlinear system (3.1) under the H_∞ performance constraint (3.17) with minimum γ, it is supposed to consider the following optimization problem

$$\min_{P>0, Q>0 \tilde{N}_1>0, \tilde{N}_2>0, \tau_1>0, \tau_2>0,} \gamma \tag{3.40}$$

subject to inequality given in (3.38) or (3.39)

Then,it conclude with the following two corollaries

Corollary 3.3.1. *Given communication channel parameters* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$, *If the optimization problem (3.40) is feasible,the observer-based controller (3.9) and (*$3.8)$ *with the controller parameters* K *and* L *can be found by LMI described in (3.38) and it will exponentially stabilize the networked nonlinear system (3.1) in mean square sense with minimum* H_∞ *Performance bound* $\gamma_{\min}$.

Corollary 3.3.2. *Given communication channel parameters* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$, *If the optimization problem (3.40) is feasible,the observer-based controller (3.9) and (*$3.8)$ *with the controller parameters* K *and* L *can be found by LMI described in (3.39) and it will exponentially stabilize the networked nonlinear system (3.1) in mean square sense with minimum* H_∞ *Performance bound* $\gamma_{\min}$.

3.4 Numerical Example

In this section, a demonstration has been held by creating a matlab code for the last created LMI in (3.38) and (3.39) . Considering a system described by (3.1) and the measurement equation (3.4) . The following given parameter has been applied in this system.

$$A = \begin{bmatrix} 0.8226 & -0.633 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 23.738 & 20.287 & 0 \end{bmatrix}$$

$$D_1 = 0.1 \quad D_2 = 0.2$$

$$f(k, x_k) = \begin{bmatrix} 0.01 \sin\ x_k^1 \\ 0.01 \sin\ x_k^2 \\ 0.01 \sin\ x_k^3 \end{bmatrix} \quad x_k = \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix} and\ \ G = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

The initial conditions of the networked nonlinear system (3.1) have been assumed as $x_0 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix}$ $\hat{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and the disturbance input has been chosen to be $\omega_k = 1/k^2$.The design for the controller (3.9) for system (3.1) has been designed, such that the H_∞ performance index was minimized.

In this example, for three cases with different random communication packet loss probabilities,It has been designed three corresponding H_∞ controllers (3.9) by theorem (3.3.1). The corresponding controller parameters and the two type of the LMI approach has been simulated with the performance index under different packet loss probabilities.

The first trail was made at $\overline{\alpha} = 0.95$ and $\overline{\beta} = 0.9$ and the following results have been taken

1- With LMI approach in (3.38)

$$K = \begin{bmatrix} 0.3739 & -0.2877 & 0 \end{bmatrix} L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0208 \end{bmatrix} \gamma_{\min.} = 3.3196$$

and the graph of their simulation has been shown in figure (3.5)

2-With LMI approach in (3.39)

$$K = \begin{bmatrix} 0.3739 & -0.2877 & 0 \end{bmatrix} L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0208 \end{bmatrix} \gamma_{\min.} = 4.9034$$

and the graph of their simulation has been shown in figure(3.2)

The second trail was made at $\overline{\alpha} = 0.7$ and $\overline{\beta} = 0.65$ and the following results have been taken:

1- With LMI approach in (3.38)

$$K = \begin{bmatrix} 0.3047 & -0.2344 & 0 \end{bmatrix} L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 3.3418$$

and the graph of their simulation has been shown in figure (3.6)

2-With LMI approach in (3.39)

$$K = \begin{bmatrix} 0.3047 & -0.2344 & 0 \end{bmatrix} L = \begin{bmatrix} 0.0070 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 4.9194$$

and the graph of their simulation has been shown in figure(3.3)

The third trail was made at $\bar{\alpha} = 0.65$ and $\bar{\beta} = 0.60$ and the following results have been taken:

1- With LMI approach in (3.38)

$$K = \begin{bmatrix} 0.2938 & -0.2261 & 0 \end{bmatrix} L = \begin{bmatrix} 0.007 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 3.3457$$

and the graph of their simulation has been shown in figure (3.7)

2-With LMI approach in (3.39)

$$K = \begin{bmatrix} 0.2938 & -0.2261 & 0 \end{bmatrix} L = \begin{bmatrix} 0.007 \\ 0.0122 \\ 0.0209 \end{bmatrix} \gamma_{\min.} = 4.9222$$

and the graph of their simulation has been shown in figure(3.4).

As shown in these graphs it was appear that the system was exponentially mean square stable. A comparison has been made between the results of these two LMI models and the model in [4] as shown in the following table 3.1

It is obvious from the table that the LMI approach in source[4] can be proper for low packet loss probability in the performance index view but it will become higher in the high

source [4]					
figure #	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
3.8	0.95	0.90	$[0.8006\ \ 0.1920\ 0.0003]$	$[0.0022\ 0.0014\ \ 0.0000]^T$	0.4574
3.9	0.7	0.65	$[0.7010\ \ 0.3162\ 0.0003]$	$[0.0104\ 0.0034\ \ 0.0000]^T$	2.2357
3.10	0.65	0.6	$[0.6903\ \ 0.3335\ 0.0002]$	$[0.0421\ 0.0129\ \ 0.0000]^T$	8.9523
new first approach (3.38)					
figure#	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
3.5	0.95	0.90	$[0.3739\ \ -0.2877\ 0.0000]$	$[0.0070\ \ 0.0122\ \ 0.0208]^T$	3.3196
3.6	0.7	0.65	$\begin{bmatrix} 0.3047 & -0.2344 & 0 \end{bmatrix}$	$[0.0070\ \ 0.0122\ \ 0.0209]^T$	3.3418
3.7	0.65	0.6	$\begin{bmatrix} 0.2938 & -0.2261 & 0 \end{bmatrix}$	$[0.0070\ \ 0.0122\ \ 0.0209]^T$	3.3457
new second approach(3.39)					
figure#	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
3.2	0.95	0.90	$[0.3739\ \ -0.2877\ 0.0000]$	$[0.0070\ \ 0.0122\ \ 0.0208]^T$	4.9034
3.3	0.7	0.65	$\begin{bmatrix} 0.3047 & -0.2344 & 0 \end{bmatrix}$	$[0.0070\ \ 0.0122\ \ 0.0209]^T$	4.9194
3.4	0.65	0.6	$\begin{bmatrix} 0.2938 & -0.2261 & 0 \end{bmatrix}$	$[0.0070\ \ 0.0122\ \ 0.0209]^T$	4.9222

TABLE 3.1. Comparsion table for the first model

probability of packet loss and the same thing in its stability. Even though the performance index in the new two approaches are higher than the LMI which is in source [4] for the low probability of packet loss, they can keep acceptable average values of it in the high probability of the packet loss.It is noticed that both of them are very near to each other in the stability behavior but the performance index of approach (3.38) is minimized more than approach (3.39). Both of them keep the balanced values of the performance for the probabilities of packet loss.

3.5 Conclusion

At the end of this chapter it can be concluded that an observer-based H_∞ controller has been designed for a class of the NNCS.The packet loss change has been described as property with Bernoulli distribution and the property of the packet loss from the sensor to the controller was different from the controller to the actuator. Certain conditions have succeeded to make the system work stably in exponentially mean-square sense. It was a type of the reproduced work of [4] with a modifications in its LMI model approach and these new modifications have succeeded to achieve the H_∞ minimization performance. The numerical example explains the effects of these new modifications on the system stability and what was their level comparing to the source [4].

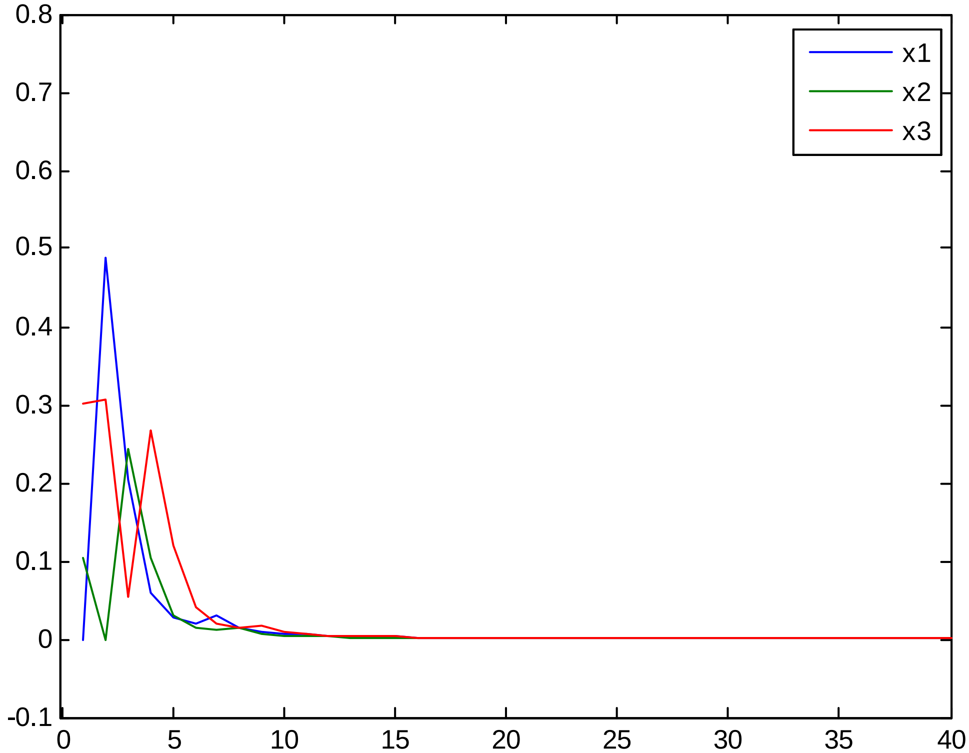

FIGURE 3.2. H_∞control for approach (3.39) at $\bar{\alpha} = .95$ and $\bar{\beta} = 0.9$

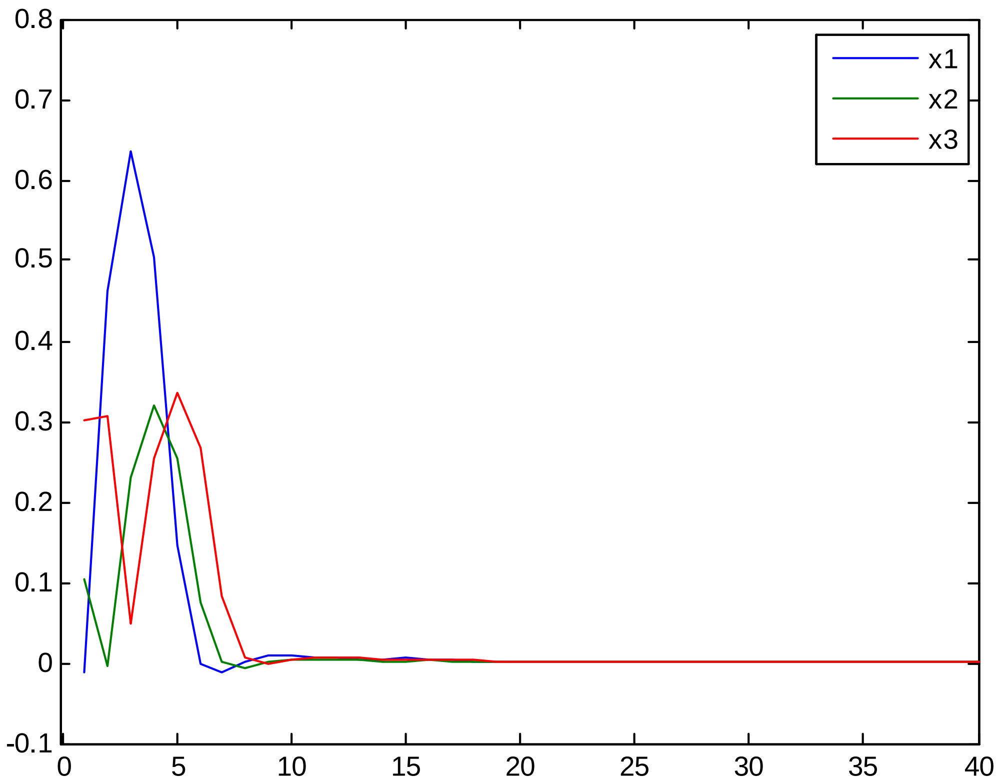

FIGURE 3.3. H_∞control for approach (3.39)at $\bar{\alpha} = .70$ and $\bar{\beta} = 0.65$

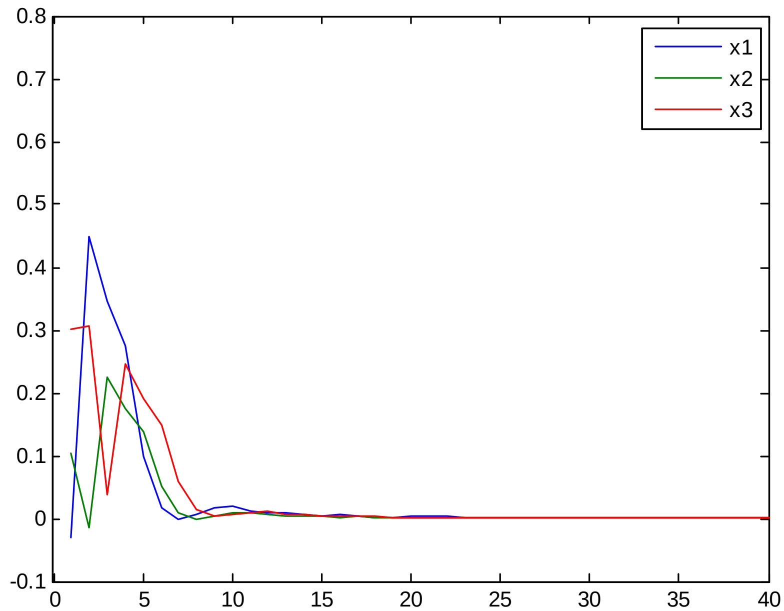

FIGURE 3.4. H_∞control for approach (3.39) at $\bar{\alpha} = .65$ and $\bar{\beta} = 0.60$

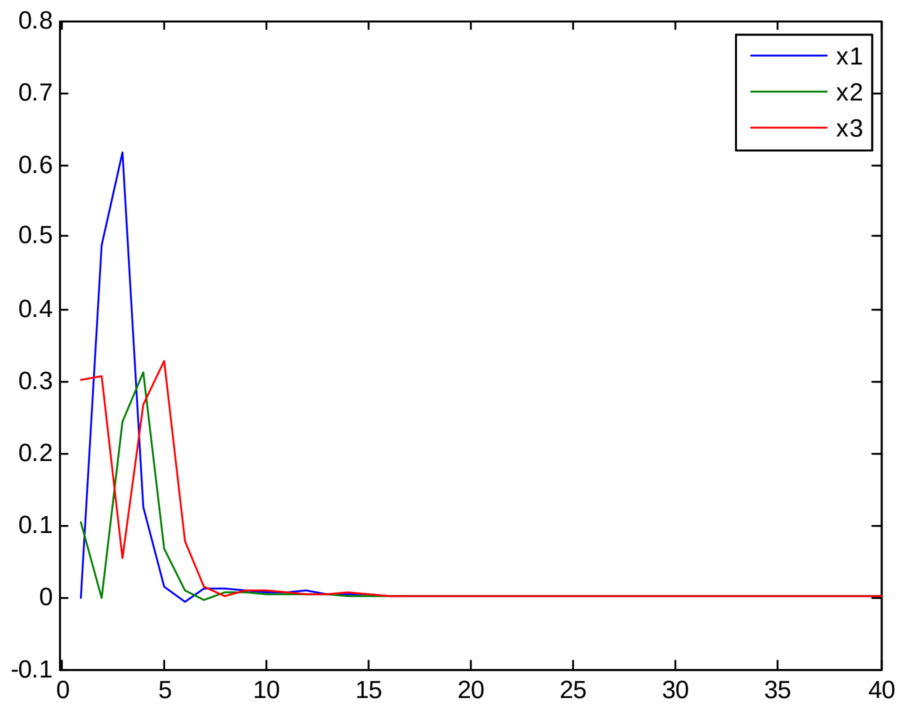

FIGURE 3.5. H_∞control for first approach (3.38) at $\bar{\alpha} = .95$ and $\bar{\beta} = 0.90$

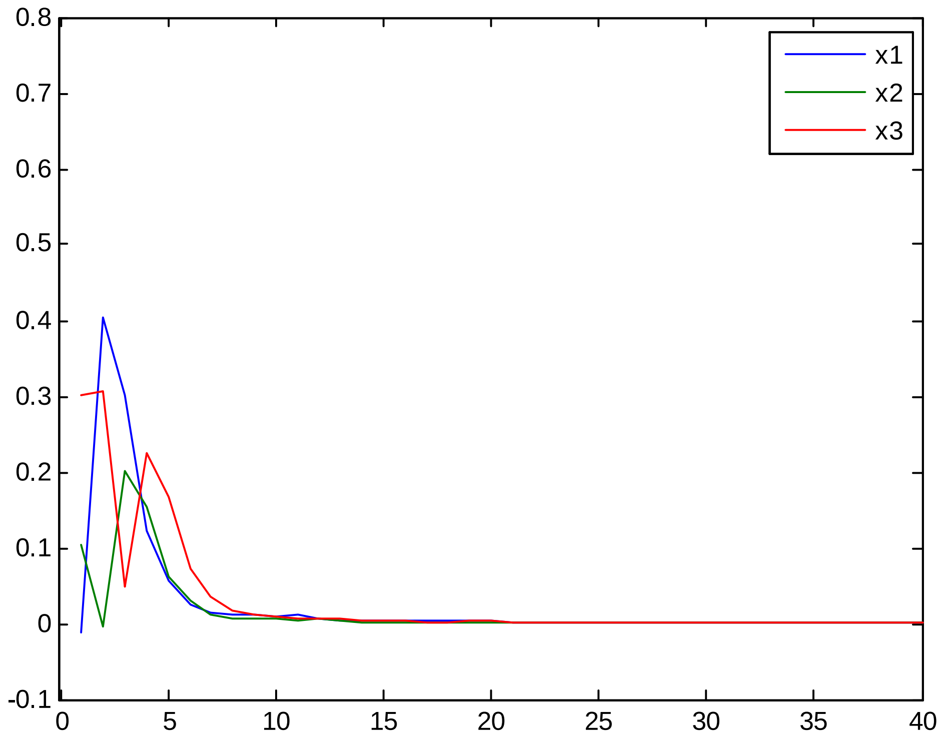

FIGURE 3.6. H_∞control for first approach (3.38) at $\bar{\alpha} = 0.7$ and $\bar{\beta} = 0.650$

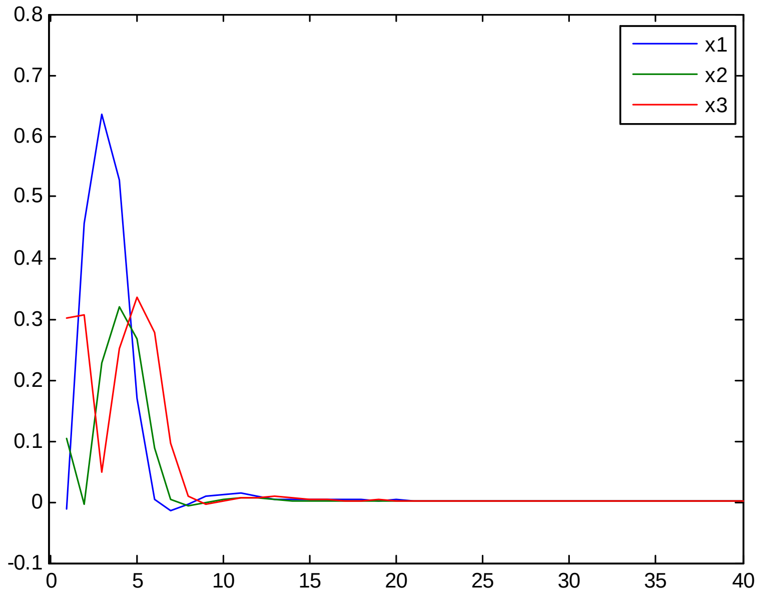

FIGURE 3.7. H_∞control for first(3.38) approach at $\bar{\alpha} = 0.65$ and $\bar{\beta} = 0.60$

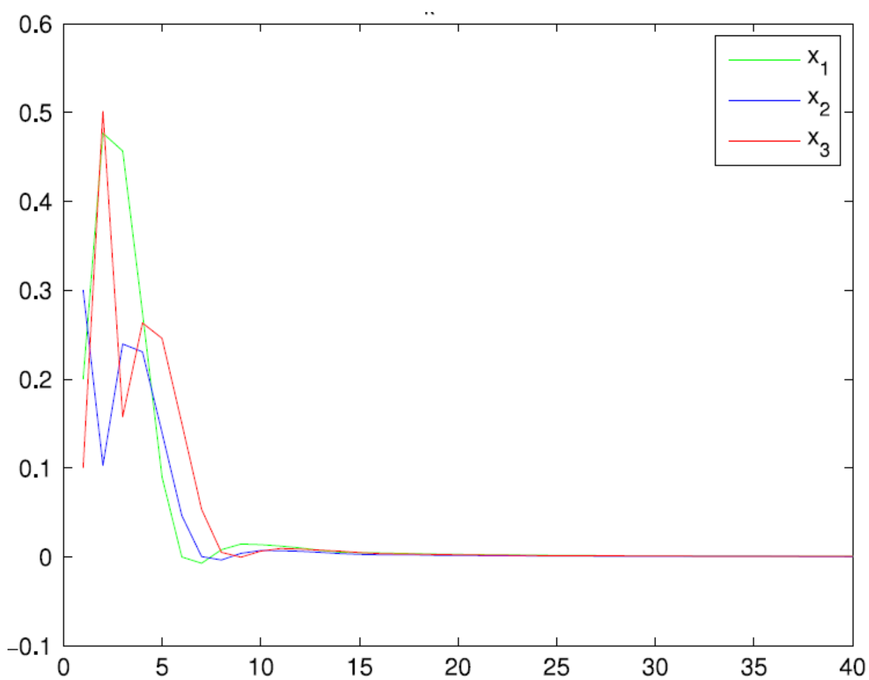

FIGURE 3.8. [4] H_∞control for LMI at $\bar{\alpha} = 0.95$ and $\bar{\beta} = 0.90$

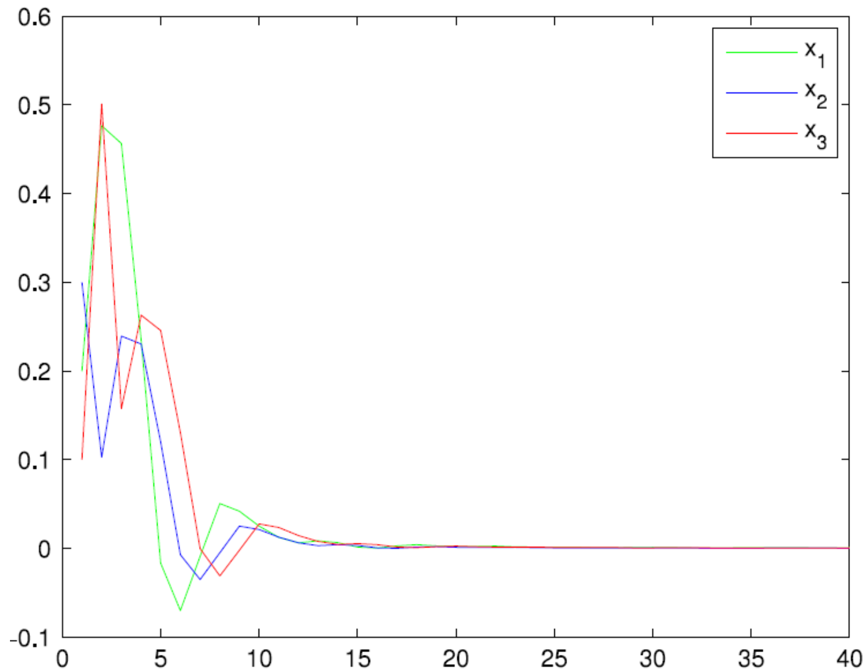

FIGURE 3.9. [4] H_∞control for LMI at $\bar{\alpha} = 0.7$ and $\bar{\beta} = 0.650$

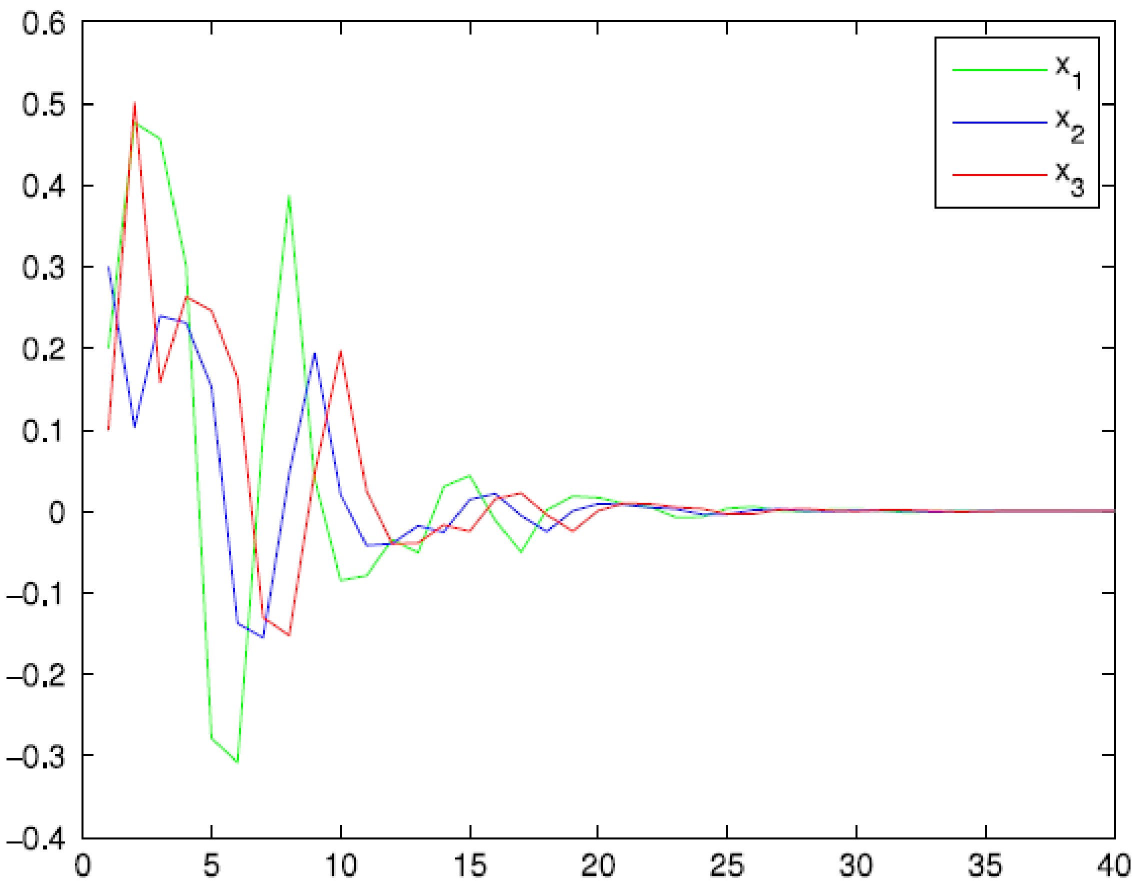

FIGURE 3.10. [4] H_∞control for LMI at $\bar{\alpha} = 0.65$ and $\bar{\beta} = 0.60$

Chapter 4

THE OBSERVER-BASED H_∞ CONTROLLER FOR UNCERTAIN NONLINEAR NCS WITH RANDOM PACKET LOSSES FIRST CLASS

4.1 Introduction

The nature of the research tilted to the simplification and to linearizing the derivations of the models for any systems while, physically, they are more complicated processes. In reality, the controlled systems are exposing to uncertainty either in signals or in their models. The existence of the noisy input or output data can cause uncertainty in the plants signals which will lead to uncertainty in the models. It was mentioned in [51] about the possible sources for the uncertainty in models of the systems. It can generally come from the following sources: inaccurate detection of the saturation in the actuators and sensors, improper failure detection for them, hardware deterioration over time and the nonlinear inherent in the physical systems that can be changed over the time. Also, the concentration in the designing does not take care for the high frequent changes in the systems. The controller is also exposed to some of the uncertainty that can come because of deliberating for reducing its order or because the imprecise implementation. It can be said that the uncertainty can be put for considering the high dynamics in the systems.

Considering the uncertainty in this thesis models has been taken from sources [5] and [50]. The authors there had applied the uncertainty in the states matrix of the systems without the considering any uncertainty in the measuring equations. In sources [5] and [50] it has been applied on linear NCS. In this thesis, uncertainty will be applied in NNCS. The NNCS that has considered here has partially some of the linear state varaibles. It has supposed that the system was considered after the sampling. The discrete system that was considered here has one transmitting data packet and the same length for its sent data packets. The delay time was not included here too. The purpose here was to find LMI approaches for one class of uncertain NNCS to design a controller based on observer with H_∞ performance.

4.2 Problem Formulation

The controlled plant discussed has been considered as a nonlinear system. The random packet losses has been considered to occur, simultaneously, in the communication channels for both way either from the sensor to the controller or from the controller to the actuator. Similar to [4], it has been supposed that the data was transmitted in a single-packet manner with the same transmission length. It has also been considered that this transmitted data ,which was employed in the point-to-point network, will have allowable data dropout rate in order to evaluate the QoS of this investigated networked nonlinear system.

Remark 4.2.1. *In this thesis, it has been supposed that obtaining the sampling of the system can be found by other ways which has not been included in this work.*

So, the considered nonlinear networked control systems here will be after the sampling as it was shown in the following:

$$\begin{aligned} x_{k+1} &= (A+\Delta A)x_k + f(k,x_k) + (B+\Delta B)u_k + Dw_k \\ z_{k+1} &= (C_1+\Delta C_1)x_k + D_1 w_k \end{aligned} \tag{4.1}$$

Where $x_k \in \mathbb{R}^n$ is the state vector,$u_k \in \mathbb{R}^m$ is the control input vector,$z_k \in \mathbb{R}^r$ is the controlled output vector,$w_k \in \mathbb{R}^q$ is the disturbance input belong to $l_2[0,\infty)$, and $A\&\Delta A \in \mathbb{R}^{n\times n}$,$B\&\Delta B \in \mathbb{R}^{n\times m}$,$D \in \mathbb{R}^{n\times q}$,$C_1\&\Delta C_1 \in \mathbb{R}^{r\times n}$,$D_1 \in \mathbb{R}^{r\times q}$ which are known constant matrices.$f(k,x_k)$ is nonlinear state varaible vector for state equation satisfies the global

Lipschitz condition:

$$\|f(k,x)\| \leqslant \|Gx\| \tag{4.2}$$

$$\|f(k,x)-f(k,y)\| \leqslant \|G(x-y)\| \tag{4.3}$$

where G is known real constant matrix with appropriate dimensions. Also, the uncertainty matrices can be described by the following limits

$$\begin{gathered}[\Delta A \;\; \Delta B\,] = M_c\,\Delta_k\,[N_1\;N_2] \\ \Delta C_1 = N_3 \Delta_k^T M_c^T\end{gathered} \tag{4.4}$$

where M_c , N_1, N_2 and N_3 are real known constant matrices with appropriate dimensions. Δ_k is unknown real matrix varying with time and has the constraint

$$\Delta_k \Delta_k^T < I \tag{4.5}$$

Remark 4.2.2. *The matrix dimensions for Δ_k ,that have been considered in the previous constraint, belonging to $\mathbb{R}^{v_i \times v_i}$(a square matrix) then the dimensions for the matrix M_c will belong to $\mathbb{R}^{n \times v_i}$ where $v_i \leq n$. If Δ_k is not a square matrix or it belongs to $\mathbb{R}^{v_i \times w_i}$ where v_i and w_i are less than or equal to n, there will be two cases. The first case is that*

$v_i < w_i$, then the constraint will be $\Delta_k^T\ \Delta_k < I$. The second case is that $w_i < v_i$, then the constraint will be $\Delta_k \Delta_k^T\ < I$.

Remark 4.2.3. *The work of this chapter has been based on the assumption that Δ_k is a square matrix belongs to $R^{v_i \times v_i}$ where $v_i \leq n$.*

The considered networked control system (4.1) is a nonlinear system. However, the networked control system that was considered in [5] was a linear system.Also, in [5] the uncertainty was not considered in the input matrix but it has been considered here. The stability analysis and controller synthesis for networked nonlinear system (4.1) with random packet losses is very important in both theory and applications, and it is also a very challenging problem. The uncertainty was considered in the linear part here for the state matrices.The feedback equation has not been considered to have the uncertainty. It was supposed that the uncertainty has been merged with its stochastic variable and it has been considered as a scalar variable.The measurment with random communication packet loss has been described by the following equation

$$\hat{y}_k = \alpha_k (C_2 x_k) + D_2 w_k \tag{4.6}$$

where $\widehat{y}_k \in \mathbb{R}^p$ is the measured output vector,$C_2 \in \mathbb{R}^{p\times n}$,$D_2 \in \mathbb{R}^{p\times q}$ are real constant matrices.The stochastic varaible $\alpha_k \in \mathbb{R}$ is a Bernoulli distributed white sequence with the following property

$$\begin{aligned} \Pr\{\alpha_k &= 1\} = \mathbb{E}x\{\alpha_k\} = \bar{\alpha} && (4.7) \\ \Pr\{\alpha_k &= 0\} = 1 - \mathbb{E}x\{\alpha_k\} = 1 - \bar{\alpha} && (4.8) \\ var\{\alpha_k\} &= \mathbb{E}x\{(\alpha_k - \bar{\alpha})^2\} = (1-\bar{\alpha})\bar{\alpha} = \alpha_1^2 && (4.9) \end{aligned}$$

The dynamic observer-based control scheme for the networked nonlinear system (4.1) is described by the following .The observer equation has been considered to be in the following form

$$\begin{gathered} \hat{x}_{k+1} = (A + \Delta A)\hat{x}_k + f(k, \hat{x}_k) + (B + \Delta B)\bar{u}_k + L(\hat{y}_k - \bar{\alpha}(C_2\hat{x}_k)) \\ \bar{u}_k = \bar{\beta}\hat{u}_k \end{gathered} \tag{4.10}$$

Controller

$$\begin{gathered} \hat{u}_k = -K\hat{x}_k \\ u_k = \beta_k\hat{u}_k \end{gathered} \tag{4.11}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state estimate of the networked non linear system (4.1),$\bar{u}_k \in \mathbb{R}^m$ is the control input of the observer,$\hat{u}_k \in \mathbb{R}^m$ is the control input without random packet loss,$u_k \in \mathbb{R}^m$ is the control input of the controlled system,$L \in \mathbb{R}^{n\times p}$ is the observer gain and $K \in \mathbb{R}^{m\times n}$is the controller gain.Also, the same thing that had happened in the previous chapter L and K are the two parameters that should be found. The stochastic variable $\beta_k \in \mathbb{R}$ is a Bernoulli distributed white sequence with the following properties.

$$\Pr\{\beta_k = 1\} = \mathbb{E}x\{\alpha_k\} = \bar{\beta} \tag{4.12}$$

$$\Pr\{\beta_k = 0\} = 1 - \mathbb{E}x\{\alpha_k\} = 1 - \bar{\beta} \tag{4.13}$$

$$var\{\beta_k\} = \mathbb{E}x\{(\beta_k - \bar{\beta})^2\} = (1-\bar{\beta})\bar{\beta} = \beta_1^2 \tag{4.14}$$

Remark 4.2.4. *α_k is the linear stochastic variable that simulate the packet dropout from the sensor to the controller. It has been assumed that it should be different than β_k which is the linear stochastic variable that simulate the packet dropout from the controller to the actuator.*

The layout of this system has been considered like the one that had been shown in figure (3.1) but the plant here should contain the uncertainty inside it. Also, the control input of the observer $\bar{u}_k$ has been consdered differenet from the control input of the controlled system u_k because of the existence of the random packet losses in the communication channel from the controller to the actuator. Now the state estimation error can be defined in the following:

$$e_k = x_k - \hat{x}_k \tag{4.15}$$

subtitute (4.6) and (4.10)-(4.11) into (4.1) and (4.15) as shown in the following. First subtitute $\hat{u}_k = -K(\hat{x}_k)$ in $\bar{u}_k = \bar{\beta}\hat{u}_k$ and in $u_k = \beta_k \hat{u}_k$

$$\begin{aligned} \bar{u}_k &= -\bar{\beta}K(\hat{x}_k) \\ u_k &= -\beta_k K(\hat{x}_k) \end{aligned}$$

Then, they were subtituted in the following equation

$$x_{k+1} = (A + \Delta A)x_k + f(k, x_k) + (B + \Delta B)u_k + Dw_k$$

This gave the following equation

$$x_{k+1} = (A + \Delta A)x_k + f(k, x_k) - \beta_k(B + \Delta B)K\hat{x}_k + Dw_k$$

From (4.15) $\hat{x}_k$ can equal $x_k - e_k$ or $\hat{x}_k = x_k - e_k$ .So

$$\begin{aligned} x_{k+1} &= (A + \Delta A)x_k + f(k, x_k) - \beta_k(B + \Delta B)K\hat{x}_k + Dw_k \\ &= ((A + \Delta A) - \beta_k(B + \Delta B)K)x_k + f(k, x_k) \\ &\quad +\beta_k(B + \Delta B)Ke_k + Dw_k \end{aligned}$$

$$\begin{aligned} &= ((A + \Delta A) - \beta_k(B + \Delta B)K)x_k + f(k, x_k) + \beta_k(B + \Delta B)Ke_k + Dw_k \\ &\quad +\bar{\beta}\left(B + \Delta B\right)Kx_k - \bar{\beta}\left(B + \Delta B\right)Kx_k + \bar{\beta}\left(B + \Delta B\right)Ke_k \\ &\quad -\bar{\beta}\left(B + \Delta B\right)Ke_k \end{aligned}$$

$$\begin{aligned} &= ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k + (\beta_k - \bar{\beta})\left(B + \Delta B\right)Ke_k \\ &\quad -(\beta_k - \bar{\beta})\left(B + \Delta B\right)Kx_k + \bar{\beta}\left(B + \Delta B\right)Ke_k + f(k, x_k) + Dw_k \end{aligned}$$

Aslo e_{k+1}has been defined with the following equation $e_{k+1} = x_{k+1} - \hat{x}_{k+1}$ or it can be written in the following form after the substitution for x_{k+1} and $\hat{x}_{k+1}$ respectively which can be shown in the following form.

$$
\begin{aligned}
e_{k+1} &= [((A+\Delta A)-\bar{\beta}(B+\Delta B)K)x_k + (\beta_k-\bar{\beta})\,(B+\Delta B)\,Ke_k \\
&\quad -(\beta_k-\bar{\beta})\,(B+\Delta B)\,Kx_k \\
&\quad +\bar{\beta}\,(B+\Delta B)\,Ke_k + f(k,x_k) + Dw_k] \\
&\quad -[\big((A+\Delta A)-\bar{\beta}(B+\Delta B)K\big)\,\hat{x}_k + f(k,\hat{x}_k) + L(\hat{y}_k-\bar{\alpha}C_2\hat{x}_k)]
\end{aligned}
$$

$$
\begin{aligned}
&= [((A+\Delta A)-\bar{\beta}(B+\Delta B)K)x_k + (\beta_k-\bar{\beta})(B+\Delta B)Ke_k \\
&\quad -(\beta_k-\bar{\beta})(B+\Delta B)Kx_k \\
&\quad +\bar{\beta}(B+\Delta B)Ke_k + f(k,x_k) + Dw_k] \\
&\quad -[(((A+\Delta A)-\bar{\beta}(B+\Delta B)K)x_k - ((A+\Delta A)-\bar{\beta}(B+\Delta B)K)e_k) \\
&\quad +f(k,\hat{x}_k) + (\alpha_k LC_2x_k + LD_2w_k - (\bar{\alpha}LC_2x_k - \bar{\alpha}LC_2e_k)]
\end{aligned}
$$

$$
\begin{aligned}
&= -(\beta_k-\bar{\beta})(B+\Delta B)Kx_k - (\alpha_k-\bar{\alpha})LC_2x_k + ((A+\Delta A)-\bar{\alpha}LC_2)e_k \\
&\quad +(\beta_k-\bar{\beta})(B+\Delta B)Ke_k + f(k,x_k) - f(k,\hat{x}_k) + (D+LD_2)w_k
\end{aligned}
$$

This has resulted in the following closed-loop netwroked nonlinear system:

$$
\begin{aligned}
x_{k+1} &= ((A+\Delta A)-\bar{\beta}(B+\Delta B)K)x_k+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k \\
&\quad -(\beta_k-\bar{\beta})(B+\Delta B)Kx_k \\
&\quad +\bar{\beta}(B+\Delta B)Ke_k+f(k,x_k)+Dw_k \\
e_{k+1} &= -(\beta_k-\bar{\beta})(B+\Delta B)Kx_k-(\alpha_k-\bar{\alpha})LC_2x_k+((A+\Delta A)-\bar{\alpha}LC_2)e_k \\
&\quad +(\beta_k-\bar{\beta})(B+\Delta B)Ke_k+F_k+(D+LD_2)w_k
\end{aligned}
\tag{4.16}
$$

where $F_k = f(k,x_k)-f(k,\hat{x}_k)$.Then, by defining

$$
\eta_k = \begin{bmatrix} x_k \\ e_k \end{bmatrix} \tag{4.17}
$$

The close nonlinear system in (4.16) can be described in the following compact form

$$
\eta_{k+1} = \bar{A}\eta_k+(\beta_k-\bar{\beta})\hat{A}_1\eta_k+(\alpha_k-\bar{\alpha})\hat{A}_2\eta_k+\bar{F}_k+\bar{B}w_k \tag{4.18}
$$

where $\bar{A} = \begin{bmatrix} (A+\Delta A)+\bar{\beta}(B+\Delta B)K & \bar{\beta}(B+\Delta B)K \\ 0 & (A+\Delta A)-\bar{\alpha}LC_2 \end{bmatrix}$,
$\hat{A}_1 = \begin{bmatrix} -(B+\Delta B)K & (B+\Delta B)K \\ -(B+\Delta B)K & (B+\Delta B)K \end{bmatrix}$, $\hat{A}_2 = \begin{bmatrix} 0 & 0 \\ -LC_2 & 0 \end{bmatrix}$, $\bar{F}_k = \begin{bmatrix} f(k,x_k) \\ F_k \end{bmatrix}$, $\bar{B} = \begin{bmatrix} D \\ D-LD_2 \end{bmatrix}$,$F_k = f(k,x_k)-f(k,\hat{x}_k)$.

This closed nonlinear system netwrok ,contains the sotchastic parameteres α_k, β_k. Then it has required to ensure its stability. The same aims ,which have been mentioned in the previous chapter, are requited here again but the uncertainty was considered here.

4.2.1 The Main Objectives

The objectives have been to design the observer (4.10) and the observer-based controller (4.11) for the networked nonlinear system (4.1), such that, in the presence the random packet losses, the closed-loop networked nonlinear system (4.18) should be sufficiently stable and the H_∞ performance constraint shuold also be satisfied.

It can be summarized ,like the previous chapter, by these two points.

1-The closed -loop nonlinear networcked system (4.16) for the system (4.18) is sufficiently exponentaily mean square stable.

2-Under the zero-intial condition , for all nonzero w_k the controlled output z_k should satisfy

$$\sum_{k=0}^{\infty} \mathbb{E} \left\| z_k \right\|^2 < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E} \left\| w_k \right\|^2 \tag{4.19}$$

where $\gamma > 0$ is a prescribed scalar.

4.3 Main Results with Proofs

The main results have been introduced in this section. Without loss of generality, it may be valued to introduce the following lemmas for technical convenience.

Lemma 4.3.1. *([4,41])(S-procedure)). Let* $T_i \in \mathbb{R}^{n\times n}(i = 0, 1, 2, ..., p)$ *be symmetric matrices. The conditions on* T_i $(i = 0, 1, 2, ..., p)$ $\varsigma^T T_o \varsigma > 0, \quad \forall\varsigma \neq 0 \; s.t. \varsigma^T T_i \varsigma \geq 0 \;\; (i = 0, 1, 2, ..., p)$ *hold if there exist* $\tau_i \geq 0$ $(i = 0, 1, 2, ..., p)$ *such that* $T_o - \sum\limits_{i=1}^{p} \tau_i T_i > 0$

Lemma 4.3.2. *([5,41]) Let* $M = M^T$ *and H and E be real matrices of appropriate dimensions with F satisfying* $FF^T < I \;\; then \;\; M + HF\mathbb{E} + \mathbb{E}^T F^T H^T < 0$, *if and only if there exists a postive scalar* $\epsilon > 0$ *such that*

$$M + \frac{1}{\epsilon} HH^T + \epsilon \mathbb{E}^T \mathbb{E} < 0 \tag{4.20}$$

or equivalently

$$\begin{bmatrix} M & H & \epsilon\mathbb{E}^T \\ H^T & -\epsilon I & 0 \\ \epsilon\mathbb{E} & 0 & -\epsilon I \end{bmatrix} < 0 \tag{4.21}$$

The same manner, which has been made in the previous chapter, has been used here but by adding the uncertainty effects on this model.

4.3.1 Finding the Matrix Inequality

In the following theorem, a derviation has been made to find a sufficient condition such that the closed-loop networked nonlinear system (4.18) is exponentialy mean square stable.

Theorem 4.3.1. *Gven the communication channel parameters* $0 \leq \bar{\alpha} \leq 1, 0 \leq \bar{\beta} \leq 1$ *, the controller gain matrix* K *and the observer gain matrix* L*. The closed-loop networked nonlinear system (4.18) is uniformly stable if there exists positive-definite matrices* P, Q*, and nonnegative real scalars* $\tau_1 > 0, \tau_2 > 0$ *with the uncertainty that was mentioned in (4.4)and (4.5) satisfying the linear matrix inequality shown below*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12}^T \\ \Pi_{12} & \Pi_{22} \end{bmatrix} < 0 \tag{4.22a}$$

where

$$\Pi_{11} = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Pi_{12} = \begin{bmatrix} P((A) - \bar{\beta}(B)K) & \bar{\beta}P(B)K & P & 0 \\ \beta_1 P(B)K & -\beta_1 P(B)K & 0 & 0 \\ \beta_1 Q(B)K & -\beta_1 Q(B)K & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 \\ 0 & Q((A) - \bar{\alpha}LC_2) & 0 & Q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{2\,2} = \begin{bmatrix} \Sigma_{11} & \Sigma_{21}^T \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix},$$

$$\Sigma_{11} = \begin{bmatrix} -P & * & * \\ 0 & -P & * \\ 0 & 0 & -Q \end{bmatrix},$$

$$\Sigma_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (PM_c)^T & \beta_1 (PM_c)^T & \beta_1 (QM_c)^T \\ \bar{\beta}(PM_c)^T & -\beta_1 (PM_c)^T & -\beta_1 (QM_c)^T \\ \epsilon(N_1 - \bar{\beta}N_2K) & \epsilon(N_2K) & \epsilon(N_2K) \\ \epsilon(N_2K) & \epsilon(N_2K) & \epsilon(N_2K) \end{bmatrix},$$

$\Sigma_{22} = \begin{bmatrix} -Q & * & * & * & * & * \\ 0 & -Q & * & * & * & * \\ 0 & 0 & -\epsilon I & * & * & * \\ 0 & (QM_c)^T & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * \\ 0 & \epsilon N_1 & 0 & 0 & 0 & -\epsilon I \end{bmatrix}$, $\alpha_1 = [(1-\bar{\alpha})\bar{\alpha}]^{1/2}$ *and* $\beta_1 = [(1-\bar{\beta})\bar{\beta}]^{1/2}$.

The same thing the following Lyapunov function has been used in the coming proof

$$V_k = x_k^T P x_k + e_k^T Q e_k \tag{4.23}$$

Proof. By using the previouse Laypunov function,where P, Q are positive definite matrices solution to

$\Delta V_k = \mathrm{E}\{V_{k+1} | x_k, x_{k-1}, x_{k-2}, ..., x_0, e_k, e_{k-1}, ..., e_0\} - V_k$

$\Delta V_k = \mathrm{E}\{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k$

$$\Delta V_k = \mathbb{E}\left\{[V_1]^T P [V_1] + [V_2]^T Q [V_2]\right\} - x_k^T P x_k - e_k^T Q e_k^T \tag{4.24}$$

where

$V_1 = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k + (\beta_k - \bar{\beta})(B + \Delta B)K e_k$

$$-(\beta_k-\bar{\beta})(B+\Delta B)Kx_k+\bar{\beta}(B+\Delta B)Ke_k+f(k,x_k)$$

$$V_2=-(\beta_k-\bar{\beta})(B+\Delta B)Kx_k-(\alpha_k-\bar{\alpha})LC_2x_k$$

$$+((A+\Delta A)-\bar{\alpha}LC_2)e_k+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k+F_k$$

For the simpilicity in deriving the equations the following to has been assumed

$$T_1=((A+\Delta A)-\bar{\beta}\,(B+\Delta B)\,K)x_k+\bar{\beta}\,(B+\Delta B)\,Ke_k+f(k,x_k)$$

$$T_2=(B+\Delta B)\,Ke_k-(B+\Delta B)\,Kx_k$$

$$T_3=((A+\Delta A)-\bar{\alpha}LC_2)e_k+F_k$$

$$T_4=LC_2x_k$$

So ΔV_k can be rewritten as following

$$\Delta V_k=\mathrm{E}\{\left[T_1+(\beta_k-\bar{\beta})T_2\right]^T$$

$$\times\left[PT_1+(\beta_k-\bar{\beta})PT_2\right]\}$$

$$+\mathrm{E}\{\left[T_3-(\alpha_k-\bar{\alpha})T_4+(\beta_k-\bar{\beta})T_2\right]^T$$

$$\times\left[QT_3-(\alpha_k-\bar{\alpha})QT_4+(\beta_k-\bar{\beta})QT_2\right]\}$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

Then, It can be rewrettin in the following form.

$$\Delta V_k=\mathrm{E}\left\{T_1^TPT_1+(\beta_k-\bar{\beta})T_2^TPT_1\right\}+\mathrm{E}\left\{(\beta_k-\bar{\beta})T_1^TPT_2+(\beta_k-\bar{\beta})^2T_2^TPT_2\right\}$$

$$+\mathrm{E}\left\{T_3^TQT_3-(\alpha_k-\bar{\alpha})T_4^TQT_3+(\beta_k-\bar{\beta})T_2^TQT_3\right\}$$

$$-\mathrm{E}\big\{(\alpha_k-\bar{\alpha})T_3^TQT_4-(\alpha_k-\bar{\alpha})^2T_4^TQT_4+(\alpha_k-\bar{\alpha})(\beta_k-\bar{\beta})T_2^TQT_4\big\}$$

$$+\mathrm{E}\big\{(\beta_k-\bar{\beta})T_3^TQT_2-(\beta_k-\bar{\beta})(\alpha_k-\bar{\alpha})T_4^TQT_2+(\beta_k-\bar{\beta})^2T_2^TQT_2\big\}$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

Also in the same way.,the properties of Bernoulli distribution and the independency properties for the different two stochastic variables α_k and β_k have been used. This has given the following results E$\{(\beta_k-\bar{\beta})\}$ =E$(\beta_k)-\bar{\beta}=\bar{\beta}-\bar{\beta}=0$, and the same thing for E$\{(\alpha_k-\bar{\alpha})\}=0$.Aslo by using the property in (4.9) and (4.14), which has given the following results E$\{(\beta_k-\bar{\beta})^2\}=\beta_1^2$ and E$\{(\alpha_k-\bar{\alpha})^2\}=\alpha_1^2$.

Then , these results have led to the following equation.

$$\Delta V_k=\left[T_1^TPT_1\right]+\left[\beta_1^2T_2^TPT_2\right]+\left[T_3^TQT_3\right]+\alpha_1^2T_4^TQT_4+\left[\beta_1^2T_2^TQT_2\right]$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

After that, a substitution has been made for the following equations in the previous equation.

$$T_1=((A+\Delta A)-\bar{\beta}\,(B+\Delta B)\,K)x_k+\bar{\beta}\,(B+\Delta B)\,Ke_k+f(k,x_k)$$

$$T_2=(B+\Delta B)\,Ke_k-(B+\Delta B)\,Kx_k$$

$$T_3=((A+\Delta A)-\bar{\alpha}LC_2)e_k+F_k$$

$$T_4=LC_2x_k$$

This has led to the following form of ΔV_k.

$$\Delta V_k = \begin{bmatrix} \left(((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k+\bar{\beta}\left(B+\Delta B\right)Ke_k+f(k,x_k)\right)^T \\ \times P\left(((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k+\bar{\beta}\left(B+\Delta B\right)Ke_k+f(k,x_k)\right) \end{bmatrix}$$

$$+\begin{bmatrix} \beta_1^2\left(\left(B+\Delta B\right)Ke_k-\left(B+\Delta B\right)Kx_k\right)^T \\ \times P\left(\left(B+\Delta B\right)Ke_k-\left(B+\Delta B\right)Kx_k\right) \end{bmatrix}$$

$$+\left[(((A+\Delta A)-\bar{\alpha}LC_2)e_k+F_k)^T Q\left(((A+\Delta A)-\bar{\alpha}LC_2)e_k+F_k\right)\right]$$

$$+\alpha_1^2\left(LC_2x_k\right)^T Q\left(LC_2x_k\right)$$

$$+\begin{bmatrix} \beta_1^2\left(\left(B+\Delta B\right)Ke_k-\left(B+\Delta B\right)Kx_k\right)^T \\ \times Q\left(\left(B+\Delta B\right)Ke_k-\left(B+\Delta B\right)Kx_k\right) \end{bmatrix}$$

$$-x_k^T Px_k$$

$$-e_k^T Qe_k^T$$

This has resulted in the following form.

$$\Delta V_k = V_1+V_2+V_3+V_4+V_5+V_6+V_7-x_k^T Px_k-e_k^T Qe_k^T$$

where

$$V_1 = (((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k)^T P((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k$$

$$+\bar{\beta}((B+\Delta B)\,Ke_k)^T P((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k)$$

$$+f(k,x_k)^T P((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k$$

$$V_2 = (((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k)^T\bar{\beta}P\left(B+\Delta B\right)Ke_k$$

$$+\bar{\beta}((B+\Delta B)\,Ke_k)^T\bar{\beta}P\left(B+\Delta B\right)Ke_k$$

$$+f(k,x_k)^T\bar{\beta}P\left(B+\Delta B\right)Ke_k$$

$$V_3=(((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)x_k)^TPf(k,x_k)$$

$$+\bar{\beta}((B+\Delta B)\,Ke_k)^TPf(k,x_k)$$

$$+f(k,x_k)^TPf(k,x_k)$$

$$V_4=\beta_1^2\left(B+\Delta B\right)Ke_k)^TP\left(B+\Delta B\right)Ke_k$$

$$-\beta_1^2\left(\left(B+\Delta B\right)Kx_k\right)^T P\left(B+\Delta B\right)Ke_k$$

$$-\beta_1^2\left(B+\Delta B\right)Ke_k)^TP\left(B+\Delta B\right)Kx_k$$

$$+\beta_1^2\left(\left(B+\Delta B\right)Kx_k\right)^T P\left(B+\Delta B\right)Kx_k$$

$$V_5=(((A+\Delta A)-\bar{\alpha}LC_2)e_k)^TQ((A+\Delta A)-\bar{\alpha}LC_2)e_k$$

$$+F_k^TQ((A+\Delta A)-\bar{\alpha}LC_2)e_k$$

$$+(((A+\Delta A)-\bar{\alpha}LC_2)e_k)^TQF_k+F_k^TQF_k$$

$$V_6=\alpha_1^2\left(LC_2x_k\right)^T\left(QLC_2x_k\right)$$

$$V_7=\beta_1^2\left(B+\Delta B\right)Ke_k)^TQ\left(B+\Delta B\right)Ke_k$$

$$-\beta_1^2\left(\left(\left(B+\Delta B\right)Kx_k\right)^T Q\left(B+\Delta B\right)Ke_k$$

$$-\beta_1^2\left(B+\Delta B\right)Ke_k)^TQ\left(B+\Delta B\right)Kx_k$$

$$+\beta_1^2\left(\left(\left(B+\Delta B\right)Kx_k\right)^T Q\left(B+\Delta B\right)Kx_k$$

This has resulted in the following matrices multiplication

$$\Delta V_k = \begin{bmatrix} x_k \\ e_k \\ f(k,x_k) \\ F_k \end{bmatrix}^T \Lambda \begin{bmatrix} x_k \\ e_k \\ f(k,x_k) \\ F_k \end{bmatrix} \triangleq \xi_k^T \Lambda \xi_k \tag{4.25}$$

where

$$\Lambda = \begin{bmatrix} \Phi_{1\,1} & * & * & * \\ \Phi_{2\,1} & \Phi_{2\,2} & * & * \\ P((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K) & \bar{\beta} P\,(B+\Delta B)\,K & P & * \\ 0 & Q((A+\Delta A) - \bar{\alpha} L C_2) & 0 & Q \end{bmatrix}$$

$$\Phi_{1\,1} = ((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K)^T P((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K)$$

$$+\beta_1^2 K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K + \beta_1^2 K^T\,(B+\Delta B)^T\,Q\,(B+\Delta B)\,K$$

$$+\alpha_1^2 C_2^T L^T Q L C_2 - P$$

$$\Phi_{2\,2} = \bar{\beta}^2\,K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K + \beta_1^2 K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K$$

$$+\beta_1^2 K^T\,(B+\Delta B)^T\,Q\,(B+\Delta B)\,K$$

$$+((A+\Delta A) - \bar{\alpha} L C_2)^T Q((A+\Delta A) - \bar{\alpha} L C_2) - Q$$

$$\Phi_{2\,1} = \bar{\beta} K^T\,(B+\Delta B)^T\,P((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K)$$

$$-\beta_1^2 K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K$$

$$-\beta_1^2 K^T\,(B+\Delta B)^T\,Q\,(B+\Delta B)\,K$$

The constraints (4.2) and (4.3) have been used which had the following expression.

$$f^T(k,x_k)f(k,x_k) = \|f(k,x)\|^2 \leqslant \|Gx\|^2 = x_k^T G^T G x_k \tag{4.26}$$

$$F_k^T F_k = \|F_k\|^2 \leqslant \|Ge_k\|^2 = e_k^T G^T G e_k \tag{4.27}$$

This has led to the following two inequalities . The first inequality was $f^T(k,x_k)f(k,x_k) \leqslant x_k^T G^T G x_k$ or$f^T(k,x_k)f(k,x_k) - x_k^T G^T G x_k \leqslant 0$. and it could be rewritten in the following matrix form.

$$\xi_k^T \begin{bmatrix} -G^T G & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_1 \xi_k \leq 0 \tag{4.28}$$

The same thing has happanded to $F_k^T F_k \leqslant e_k^T G^T G e_k$ or $F_k^T F_k - e_k^T G^T G e_k \leqslant 0$ and it has resulted in the following matrix form.

$$\xi_k^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -G^T G & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_2 \xi_k \leq 0 \tag{4.29}$$

By the Lemma (4.3.1) with the constrains (4.28) and (4.29) for $\Delta V_k = \xi_k^T \Lambda \xi_k < 0$, it holds if there exists positive-definite matricesP, Q and positive scalars $\tau_1 > 0, \tau_2 > 0$ such that

$$\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 < 0 \tag{4.30}$$

then $\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2$

$$= \begin{bmatrix} \tilde{\Phi}_{1\,1} & * & * & * \\ \tilde{\Phi}_{2\,1} & \tilde{\Phi}_{2\,2} & * & * \\ \tilde{\Phi}_{3\,1} & \bar{\beta}P\left(B+\Delta B\right)K & P-\tau_1 I & * \\ 0 & Q((A+\Delta A)-\bar{\alpha}LC_2) & 0 & Q-\tau_2 I \end{bmatrix} < 0$$

where

$$\tilde{\Phi}_{1\,1} = ((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)^T P((A+\Delta A)$$

$$-\bar{\beta}\left(B+\Delta B\right)K) + \beta_1^2 K^T\left(B+\Delta B\right)^T P\left(B+\Delta B\right)K$$

$$+\beta_1^2 K^T\left(B+\Delta B\right)^T Q\left(B+\Delta B\right)K + \alpha_1^2 C_2^T L^T Q L C_2$$

$$-P + \tau_1 G^T G^T$$

$$\tilde{\Phi}_{2\,2} = \bar{\beta}^2 K^T\left(B+\Delta B\right)^T P\left(B+\Delta B\right)K + \beta_1^2 K^T\left(B+\Delta B\right)^T P\left(B+\Delta B\right)K$$

$$+\beta_1^2 K^T\left(B+\Delta B\right)^T Q\left(B+\Delta B\right)K$$

$$+((A+\Delta A)-\bar{\alpha}LC_2)^T Q((A+\Delta A)-\bar{\alpha}LC_2)$$

$$-Q + \tau_2 G^T G^T$$

$$\tilde{\Phi}_{2\,1} = \bar{\beta}K^T\left(B+\Delta B\right)^T P((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K)$$

$$-\beta_1^2 K^T\left(B+\Delta B\right)^T P\left(B+\Delta B\right)K$$

$$-\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$\tilde{\Phi}_{3\ 1} = P((A+\Delta A) - \bar{\beta} (B+\Delta B) K)$$

It can be rewritten in the equivalent form of a MI as it has been shown in the following.

$$\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 = \begin{bmatrix} -P+\tau_1 G^T G^T & * & * & * \\ 0 & -Q+\tau_2 G^T G^T & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$$

$$+\begin{bmatrix} \breve{\Phi}_{11} & * & * & 0 \\ \breve{\Phi}_{21} & \breve{\Phi}_{22} & * & * \\ P((A+\Delta A) - \bar{\beta} (B+\Delta B) K) & \bar{\beta} P (B+\Delta B) K & P & 0 \\ 0 & Q((A+\Delta A) - \bar{\alpha} L C_2) & 0 & Q \end{bmatrix} < 0$$

where

$$\breve{\Phi}_{11} = ((A+\Delta A) - \bar{\beta} (B+\Delta B) K)^T P((A+\Delta A) - \bar{\beta} (B+\Delta B) K)$$

$$+\beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K + \beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$+\alpha_1^2 C_2^T L^T Q L C_2$$

$$\breve{\Phi}_{21} = \bar{\beta} K^T (B+\Delta B)^T P((A+\Delta A) - \bar{\beta} (B+\Delta B) K)$$

$$-\beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$-\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$\breve{\Phi}_{22} = \bar{\beta}^2 K^T (B+\Delta B)^T P (B+\Delta B) K + \beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$+\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$+((A+\Delta A)-\bar{\alpha}LC_2)^T Q((A+\Delta A)-\bar{\alpha}LC_2)$$

It can be rewritten in the following form $\Lambda-\tau_1\Lambda_1-\tau_2\Lambda_2=\begin{bmatrix}\tilde{\Lambda}_1 & \Psi_1^T \\ \Psi_1 & \breve{\Lambda}_1\end{bmatrix}<0$

where

$$\tilde{\Lambda}_1=\begin{bmatrix}-P+\tau_1 G^T G^T & * & * & * \\ 0 & -Q+\tau_2 G^T G^T & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I\end{bmatrix},$$

$$\breve{\Lambda}_1=\begin{bmatrix}-P & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & -Q\end{bmatrix},$$

$$\Psi_1=\begin{bmatrix}P((A+\Delta A)-\bar{\beta}\left(B+\Delta B\right)K) & \bar{\beta}P\left(B+\Delta B\right)K & P & 0 \\ \beta_1 P\left(B+\Delta B\right)K & -\beta_1 P\left(B+\Delta B\right)K & 0 & 0 \\ \beta_1 Q\left(B+\Delta B\right)K & -\beta_1 Q\left(B+\Delta B\right)K & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 \\ 0 & Q((A+\Delta A)-\bar{\alpha}LC_2) & 0 & Q\end{bmatrix}.$$

To separate the elements that contain Δ matrices, the previous inequality can be rewirtten in the following form.

$$\Lambda-\tau_1\Lambda_1-\tau_2\Lambda_2=\begin{bmatrix}\tilde{\Lambda}_2 & \Psi_2^T \\ \Psi_2 & \breve{\Lambda}_2\end{bmatrix}+\begin{bmatrix}\tilde{\Lambda}_3 & \Psi_3^T \\ \Psi_3 & \breve{\Lambda}_3\end{bmatrix}$$

where

$$\tilde{\Lambda}_2 = \begin{bmatrix} -P+\tau_1 G^T G^T & * & * & * \\ 0 & -Q+\tau_2 G^T G^T & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\breve{\Lambda}_2 = \begin{bmatrix} -P & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} P((A)-\bar{\beta}\,(B)\,K) & \bar{\beta}P\,(B)\,K & P & 0 \\ \beta_1 P\,(B)\,K & -\beta_1 P\,(B)\,K & 0 & 0 \\ \beta_1 Q\,(B)\,K & -\beta_1 Q\,(B)\,K & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 \\ 0 & Q((A)-\bar{\alpha}LC_2) & 0 & Q \end{bmatrix},$$

$$\tilde{\Lambda}_3 = \begin{bmatrix} 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\breve{\Lambda}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_3 = \begin{bmatrix} (P(\Delta A) - \bar{\beta} P(\Delta B) K) & \bar{\beta} P(\Delta B) K & 0 & 0 \\ \beta_1 P(\Delta B) K & -\beta_1 P(\Delta B) K & 0 & 0 \\ \beta_1 Q(\Delta B) K & -\beta_1 Q(\Delta B) K & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & Q(\Delta A) & 0 & 0 \end{bmatrix}$$

Based on (4.4) with the two Lemmas (4.20) and (4.21), it can be rewritten in the following form.

$$\Lambda - \tau_1 \Lambda_1 - \tau_2 \Lambda_2 =$$
$$\begin{bmatrix} \tilde{\Lambda}_6 & \Psi_6^T \\ \Psi_6 & \breve{\Lambda}_6 \end{bmatrix} + [\Psi_9] \begin{bmatrix} \Delta_k & 0 \\ 0 & \Delta_k \end{bmatrix} [\Psi_{10}] + [\Psi_{10}]^T \begin{bmatrix} \Delta_k^T & 0 \\ 0 & \Delta_k^T \end{bmatrix} [\Psi_9]^T$$

where

$$\tilde{\Lambda}_6 = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\breve{\Lambda}_6 = \begin{bmatrix} -P & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\Psi_6 = \begin{bmatrix} P((A) - \bar{\beta}(B)K) & \bar{\beta}P(B)K & P & 0 \\ \beta_1 P(B)K & -\beta_1 P(B)K & 0 & 0 \\ \beta_1 Q(B)K & -\beta_1 Q(B)K & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 \\ 0 & Q((A) - \bar{\alpha}LC_2) & 0 & Q \end{bmatrix},$$

$$\Psi_9 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ (PM_c) & \bar{\beta}(PM_c) \\ \beta_1(PM_c) & -\beta_1(PM_c) \\ \beta_1(QM_c) & -\beta_1(QM_c) \\ 0 & 0 \\ 0 & (QM_c) \end{bmatrix},$$

$$\Psi_{10} = \begin{bmatrix} \left(N_1 - \bar{\beta}N_2K\right) & (N_2K) & (N_2K) & 0 & 0 & 0 & 0 & 0 & 0 \\ (N_2K) & N_2K & (N_2K) & 0 & N_1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

0 is the zero matrix with proper dimensions..

By using the constraint (4.5) $(\Delta_k \Delta_k^T) < I$, it holds if there is $\epsilon > 0$. As results of that we have the following form.

$$\Lambda - \tau_1\Lambda_1 - \tau_2\Lambda_2 = \begin{bmatrix} \tilde{\Lambda}_8 & \Psi_{11}^T \\ \Psi_{11} & \breve{\Lambda}_8 \end{bmatrix} < 0$$

where

$$\tilde{\Lambda}_8 = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * \\ 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\breve{\Lambda}_8 = \begin{bmatrix} \tilde{\Sigma} & \hat{\Psi}^T \\ \hat{\Psi} & \breve{\Sigma} \end{bmatrix},$$

$$\tilde{\Sigma} = \begin{bmatrix} -P & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & -Q \end{bmatrix}$$

$$\breve{\Sigma} = \begin{bmatrix} -\epsilon I & 0 & 0 & 0 \\ 0 & -\epsilon I & 0 & 0 \\ 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & -\epsilon I \end{bmatrix}$$

$$\hat{\Psi} = \begin{bmatrix} (PM_c)^T & \beta_1 (PM_c)^T & \beta_1 (QM_c)^T & 0 & 0 \\ \bar{\beta} (PM_c)^T & -\beta_1 (PM_c)^T & -\beta_1 (QM_c)^T & 0 & (QM_c)^T \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon N_1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Psi_{11} = \begin{bmatrix} P((A) - \bar{\beta}(B)K) & \bar{\beta}P(B)K & P & 0 \\ \beta_1 P(B)K & -\beta_1 P(B)K & 0 & 0 \\ \beta_1 Q(B)K & -\beta_1 Q(B)K & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 \\ 0 & Q((A) - \bar{\alpha}LC_2) & 0 & Q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \epsilon(N_1 - \bar{\beta}N_2K) & \epsilon(N_2K) & \epsilon(N_2K) & 0 \\ \epsilon(N_2K) & \epsilon(N_2K) & \epsilon(N_2K) & 0 \end{bmatrix},$$

This has given the same MI shown in (4.22a). Thus, the stability concluded with the following steps.

$\Delta V_k = \xi_k^T \Lambda \xi_k < 0$ If $\Lambda < 0$ and

$\Delta V_k = \xi_k^T \Lambda \xi_k \leq -\lambda_{\min}(-\Lambda)\xi_k^T \xi_k$or

$\Delta V_k \leq -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + f(k, x_k)^T f(k, x_k) + F_k^T F_k)$, then

$\Delta V_k \leq -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + \|f(k, x_k)\|^2 + \|F_k\|^2) < -\alpha \eta_k^T \eta_k$

where

$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \sigma\}$

$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \max\{\lambda_{\max}(P), \lambda_{\max}(Q)\}\}$

So, it has been proved that

$$\Delta V_k < -\alpha\eta_k^T\eta_k < -\frac{\alpha}{\sigma}V_k := -\psi V_k \tag{4.31}$$

Therefore by the definition (3.18) , it can be verified from the Theorem (1.3.1) that the closed-loop nonlinear networked system (4.16) is exponentaily mean square stable. This is the end of the proof. □

Next, the sufficient conditions have been searched such that not only the closed-loop networked nonlinear system (4.18) was exponentially stable, but also the H_∞ performance constraint (4.19) should be achieved. The constraint (4.19) has been used to describe the H_∞ performance of the stochastic nonlinear system (4.18), where the expectation operator has been utilized on both the controlled output and the disturbance input.

Theorem 4.3.2. *Given communication channel parameters* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$,*and a scalar* $\gamma > 0$. *The uncertain closed loop netwroked nonlinear system (4.18) is exponentialy mean square stable and the* H_∞ *performance constraint (4.19) is achived for all non zero* w_k, *if there exist positive definite matrices* $P > 0,\ Q > 0$ *,real matrices* K, *and* L *; and positive real scalar* $\tau_1 > 0, \tau_2 > 0, \epsilon > 0$ *satisfying the matrix inequalty* *shown in (4.32)*

$$\begin{bmatrix} \tilde{\Gamma} & \Xi^T \\ \Xi & \breve{\Gamma} \end{bmatrix} < 0 \tag{4.32}$$

where

$$\tilde{\Gamma} = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Xi = \begin{bmatrix} \Xi_{11} \\ \Xi_{21} \\ \Xi_{31} \end{bmatrix},$$

$$\Xi_{11} = \begin{bmatrix} PA - \bar{\beta} PBK & \bar{\beta} PBK & PD & P & 0 \\ \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 \\ \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 \\ 0 & QA - \bar{\alpha} QLC_2 & Q(D - LD_2) & 0 & Q \end{bmatrix},$$

$$\Xi_{21} = \begin{bmatrix} C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_{31} = \begin{bmatrix} \epsilon\left(N_1\right) & 0 & 0 & 0 & 0 \\ \epsilon\left(N_2K\right) & \epsilon\left(N_2K\right) & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 \end{bmatrix},$$

$$\breve{\Gamma} = \begin{bmatrix} -P & 0 & 0 & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -P & 0 & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & -Q & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -Q & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -I & 0 & * & * & * & * & * & * & * \\ u_{12\ 6} & 0 & 0 & 0 & 0 & 0 & u & * & * & * & * & * & * & * \\ u_{13\ 6} & u_{13\ 7} & u_{13\ 8} & 0 & 0 & 0 & 0 & u & 0 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & N_3 & 0 & 0 & u & * & * & * & * & * \\ 0 & 0 & 0 & 0 & u_{15\ 10} & 0 & 0 & 0 & 0 & u & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u \end{bmatrix},$$

$$u_{12\ 6} = \left(PM_c\right)^T$$

$$u_{13\ 6} = -\bar{\beta}\left(PM_c\right)^T$$

$$u_{13\ 7} = \beta_1\left(PM_c\right)^T$$

$$u_{13\ 8} = \beta_1\left(QM_c\right)^T$$

$$u_{15\ 10} = \left(QM_c\right)^T$$

$u = -\epsilon I$

$\alpha_1 = \left[(1-\bar{\alpha})\bar{\alpha}\right]^{1/2}$and $\beta_1 = \left[(1-\bar{\beta})\bar{\beta}\right]^{1/2}$.

Proof. For any nonzero w_k, the following equation has been derived from (4.30) .

$\mathrm{E}\{V_{k+1}\}-\mathrm{E}\{V_k\}+\mathrm{E}\{Z_k^T Z_k\} - \gamma^2 \mathrm{E}\{w_k^T w_k\}$

$=\mathrm{E}\Big\{[U_1]^T P\,[U_1] + [U_2]^T Q\,[U_2]\Big\}$

$-x_k^T P x_k - e_k^T Q e_k^T + [U_3]^T[U_3] - \gamma^2 w_k^T w_k^T \triangleq \xi_k^T \Omega \xi_k$

where

$U_1 = ((A+\Delta A) - \bar{\beta}(B-\Delta B)K)x_k + (\beta_k - \bar{\beta})(B+\Delta B)Ke_k$

$\quad -(\beta_k - \bar{\beta})(B+\Delta B)Kx_k + \bar{\beta}(B+\Delta B)Ke_k + f(k,x_k) + Dw_k$

$U_2 = -(\beta_k - \bar{\beta})(B+\Delta B)Kx_k - (\alpha_k - \bar{\alpha})LC_2 x_k$

$\quad +((A+\Delta A) - \bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})(B+\Delta B)Ke_k$

$\quad +F_k + (D - LD_2)w_k$

$U_3 = (C_1 + \Delta C_1)x_k + D_1 w_k$

For the simpilicity in deriving the equations the following assumptions have been made.

$T_1 = ((A+\Delta A) - \bar{\beta}(B+\Delta B)K)x_k$

$\quad +\bar{\beta}(B+\Delta B)Ke_k + f(k,x_k) + Dw_k$

$T_2 = (B+\Delta B)Ke_k - (B+\Delta B)Kx_k$

$T_3 = ((A + \Delta A) - \bar{\alpha} LC_2)e_k + F_k + (D - LD_2)w_k$

$T_4 = LC_2 x_k$

So $\mathrm{E}\{V_{k+1}\} - \mathrm{E}\{V_k\} + \mathrm{E}\{Z_k^T Z_k\} - \gamma^2 \mathrm{E}\{w_k^T w_k\}$ can be rewritten in the following form

$$
\begin{aligned}
&\mathrm{E}\{V_{k+1}\} - \mathrm{E}\{V_k\} + \mathrm{E}\{Z_k^T Z_k\} - \gamma^2 \mathrm{E}\{w_k^T w_k\} \\
&\quad = \mathrm{E}\big\{T_1^T P T_1 + (\beta_k - \bar{\beta}) T_2^T P T_1\big\} \\
&\quad + \mathrm{E}\big\{(\beta_k - \bar{\beta}) T_1^T P T_2 + (\beta_k - \bar{\beta})^2 T_2^T P T_2\big\} \\
&\quad + \mathrm{E}\big\{T_3^T Q T_3 - (\alpha_k - \bar{\alpha}) T_4^T Q T_3 + (\beta_k - \bar{\beta}) T_2^T Q T_3\big\} \\
&\quad - \mathrm{E}\big\{(\alpha_k - \bar{\alpha}) T_3^T Q T_4 - (\alpha_k - \bar{\alpha})^2 T_4^T Q T_4\big\} \\
&\quad - \mathrm{E}\big\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta}) T_2^T Q T_4\big\} \\
&\quad + \mathrm{E}\big\{(\beta_k - \bar{\beta}) T_3^T Q T_2 - (\beta_k - \bar{\beta})(\alpha_k - \bar{\alpha}) T_4^T Q T_2\big\} \\
&\quad + \mathrm{E}\big\{(\beta_k - \bar{\beta})^2 T_2^T Q T_2\big\} \\
&\quad - x_k^T P x_k \\
&\quad - e_k^T Q e_k^T \\
&\quad + [(C_1 + \Delta C_1)x_k + D_1 w_k]^T [(C_1 + \Delta C_1)x_k + D_1 w_k] \\
&\quad - \gamma^2 w_k^T w_k^T
\end{aligned}
$$

By using the Bernoulli distribution properties (4.9) and (4.14) with the independency properties, we have.

$$\mathrm{E}\{V_{k+1}\}-\mathrm{E}\{V_k\}+\mathrm{E}\{Z_k^T Z_k\} - \gamma^2\mathrm{E}\{w_k^T w_k\}$$

$$= \left[T_1^T P T_1\right] + \left[\beta_1^2 T_2^T P T_2\right] + \left[T_3^T Q T_3\right]$$

$$+\alpha_1^2 T_4^T Q T_4 + \left[\beta_1^2 T_2^T Q T_2\right]$$

$$-x_k^T P x_k - e_k^T Q e_k^T$$

$$+[(C_1 + \Delta C_1)x_k + D_1 w_k]^T[(C_1 + \Delta C_1)x_k + D_1 w_k]$$

$$-\gamma^2 w_k^T w_k^T$$

After that, substitutions have been made for the following assumptions.

$$T_1 = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k + \bar{\beta}(B + \Delta B)Ke_k + f(k, x_k) + Dw_k$$

$$T_2 = (B + \Delta B)Ke_k - (B + \Delta B)Kx_k$$

$$T_3 = ((A + \Delta A) - \bar{\alpha}LC_2)e_k + F_k + (D - LD_2)w_k$$

$$T_4 = LC_2 x_k$$

This has given the following form

$$\mathrm{E}\{V_{k+1}\}-\mathrm{E}\{V_k\}+\mathrm{E}\{Z_k^T Z_k\} - \gamma^2\mathrm{E}\{w_k^T w_k\}$$

$$= (((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k)^T \times P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k$$

$$+\bar{\beta}((B + \Delta B)Ke_k)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k$$

$$+f(k, x_k)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k$$

$$+(Dw_k)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k$$

$$+(((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k)^T \bar{\beta} P(B + \Delta B)Ke_k$$

$$+\bar{\beta}((B + \Delta B)Ke_k)^T \bar{\beta} P(B + \Delta B)Ke_k$$

$$+f(k, x_k)^T \bar{\beta} P(B + \Delta B)Ke_k$$

$$+(Dw_k)^T \bar{\beta} P(B + \Delta B)Ke_k$$

$$+(((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k)^T Pf(k, x_k)$$

$$+\bar{\beta}((B + \Delta B)Ke_k)^T Pf(k, x_k)$$

$$+f(k, x_k)^T Pf(k, x_k)$$

$$+(Dw_k)^T Pf(k, x_k)$$

$$+(((A + \Delta A) - \bar{\beta}(B + \Delta B)K)x_k)^T PDw_k$$

$$+\bar{\beta}((B + \Delta B)Ke_k)^T PDw_k + f(k, x_k)^T PDw_k$$

$$+(Dw_k)^T PDw_k$$

$$+\beta_1^2 \left((B + \Delta B)Ke_k)^T P(B + \Delta B)Ke_k\right)^T$$

$$-\left(\beta_1^2((B + \Delta B)Kx_k\right)^T \times P(B + \Delta B)Ke_k$$

$$-\beta_1^2(B + \Delta B)Ke_k)^T P\left((B + \Delta B)Kx_k\right)^T$$

$$+\beta_1^2\left((B + \Delta B)Kx_k\right)^T \times P(B + \Delta B)Kx_k$$

$$+(((A - \Delta A) - \bar{\alpha}LC_2)e_k)^T \times Q((A - \Delta A) - \bar{\alpha}LC_2)e_k$$

$$+F_k^T Q((A + \Delta A) - \bar{\alpha}LC_2)e_k$$

$$+((D-LD_2)w_k)^TQ((A+\Delta A)-\bar{\alpha}LC_2)e_k$$

$$+(((A+\Delta A)-\bar{\alpha}LC_2)e_k)^TQF_k$$

$$+F_k^TQF_k+((D-LD_2)w_k)^TQF_k$$

$$+(((A+\Delta A)-\bar{\alpha}LC_2)e_k)^TQ(D-LD_2)w_k$$

$$+F_k^TQ(D-LD_2)w_k+((D-LD_2)w_k)^TQ(D-LD_2)w_k$$

$$+\alpha_1^2\left(LC_2x_k\right)^T\left(QLC_2x_k\right)$$

$$+\beta_1^2(B+\Delta B)Ke_k)^TQ(B+\Delta B)Ke_k$$

$$-\beta_1^2((B+\Delta B)Kx_k^T\times Q(B+\Delta B)Ke_k$$

$$-\beta_1^2(B+\Delta B)Ke_k)^TQ(B+\Delta B)Kx_k$$

$$+\beta_1^2(B+\Delta B)Kx_k^T\times Q(B+\Delta B)Kx_k$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

$$+((C_1+\Delta C_1)x_k)^T(C_1+\Delta C_1)x_k$$

$$+((C_1+\Delta C_1)x_k)^TD_1w_k$$

$$+(D_1w_k)^T(C_1+\Delta C_1)x_k+(D_1w_k)^TD_1w_k$$

$$-\gamma^2w_k^Tw_k^T$$

It has been constructed in a matrix shape gevin in the following form.

$$
\begin{gathered}
\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = \\
\begin{bmatrix} x_k \\ e_k \\ w_k \\ f(k, x_k) \\ F_k \end{bmatrix}^T
\begin{bmatrix}
\Omega_{1\,1} & * & * & * & * \\
\Omega_{2\,1} & \Omega_{2\,2} & * & * & * \\
\Omega_{3\,1} & \Omega_{3\,2} & \Omega_{3\,3} & * & * \\
\Omega_{4\,1} & \bar{\beta} P(B - \Delta B)K & PD & P & * \\
0 & \Omega_{5\,2} & Q(D - LD_2) & 0 & Q
\end{bmatrix}
\begin{bmatrix} x_k \\ e_k \\ w_k \\ f(k, x_k) \\ F_k \end{bmatrix} \\
\triangleq \zeta_k^T \Omega \zeta_k
\end{gathered}
\tag{4.33}
$$

where

$$
\begin{aligned}
\Omega_{1\,1} = {} & ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K) \\
& + \beta_1^2 K^T (B + \Delta B)^T P(B + \Delta B)K \\
& + \beta_1^2 K^T (B + \Delta B)^T Q(B + \Delta B)K + \alpha_1^2 C_2^T L^T Q L C_2 \\
& + (C_1 + \Delta C_1)^T (C_1 + \Delta C_1) - P \\
\Omega_{2\,2} = {} & \bar{\beta}^2 K^T (B + \Delta B)^T P(B + \Delta B)K \\
& + \beta_1^2 K^T (B + \Delta B)^T P(B + \Delta B)K \\
& + \beta_1^2 K^T (B + \Delta B)^T Q(B + \Delta B)K \\
& + ((A + \Delta A) - \bar{\alpha} L C_2)^T Q((A + \Delta A) - \bar{\alpha} L C_2) \\
& - Q
\end{aligned}
$$

$$\Omega_{2\,1} = \bar{\beta}K^T(B+\Delta B)^T P((A+\Delta A) - \bar{\beta}(B+\Delta B)K)$$

$$-\beta_1^2 K^T(B+\Delta B)^T P(B+\Delta B)K$$

$$-\beta_1^2 K^T(B+\Delta B)^T Q(B+\Delta B)K$$

$$\Omega_{3\,1} = D^T P((A+\Delta A) - \bar{\beta}(B+\Delta B)K) + D_1^T(C_1+\Delta C_1)$$

$$\Omega_{3\,2} = \bar{\beta}D^T P(B+\Delta B)K + (D-LD_2)^T Q((A+\Delta A) - \bar{\alpha}LC_2)$$

$$\Omega_{3\,3} = D^T PD + (D-LD_2)^T Q(D-LD_2) + D^T D_1 - \gamma^2 I$$

$$\Omega_{4\,1} = P(A+\Delta A) - \bar{\beta}P(B+\Delta B)K$$

$$\Omega_{5\,2} = Q((A+\Delta A) - \bar{\alpha}LC_2)$$

Also with the same manner, the constraints (4.2) and (4.3) have been used which was expressed by the following inequality.

$$f^T(k,x_k)f(k,x_k) = \|f(k,x)\|^2 \leqslant \|Gx\|^2 = x_k^T G^T G x_k \tag{4.34}$$

$$F_k^T F_k = \|F_k\|^2 \leqslant \|Ge_k\|^2 = e_k^T G^T G e_k \tag{4.35}$$

They has been reshaped in the following two MIs. The first MI got the following form.

$$\zeta_k^T \begin{bmatrix} -G^TG & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Omega_1 \zeta_k \leq 0 \tag{4.36}$$

It has resulted from $f^T(k, x_k)f(k, x_k) \leqslant x_k^T G^T G x_k$ or $f^T(k, x_k)f(k, x_k) - x_k^T G^T G x_k \leqslant 0$.

The second MI has been shaped into the following form.

$$\zeta_k^T \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -G^TG & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Omega_2 \zeta_k \leq 0 \tag{4.37}$$

It has been created from $F_k^T F_k \leqslant e_k^T G^T G e_k$ or $F_k^T F_k - e_k^T G^T G e_k \leqslant 0.$

ζ_k is the vector that contains the vectors varaibles or $\zeta_k = \begin{bmatrix} x_k \\ e_k \\ w_k \\ f(k, x_k) \\ F_k \end{bmatrix}$.

By the S-procedure, $\mathrm{E}\{V_{k+1}\} - \mathrm{E}\{V_k\} + \mathrm{E}\{Z_k^T Z_k\} - \gamma^2 \mathrm{E}\{w_k^T w_k\} = \zeta_k^T \Omega \zeta_k < 0$ with the constraints (4.28) and (4.29) holds if there exists positive-definite matrices P, Q and nonnegative scalars $\tau_1 > 0, \tau_2 > 0$ such that

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2 < 0 \tag{4.38}$$

It can be described by the following.

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2$$
$$= \begin{bmatrix} \tilde{\Omega} & * & * & * & * \\ \Omega_2 & \breve{\Omega} & * & * & * \\ \Omega_4 & \Omega_3 & \hat{\Omega} & * & * \\ \Omega_5 & \bar{\beta}P(B - \Delta B)K & PD & P + \tau_1 I & * \\ 0 & \Omega_6 & Q(D - LD_2) & 0 & Q + \tau_2 I \end{bmatrix} < 0$$

where

$$\tilde{\Omega} = ((A + \Delta A) - \bar{\beta}(B - \Delta B)K)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)$$
$$+\beta_1^2 K^T(B + \Delta B)^T P(B + \Delta B)K$$
$$+\beta_1^2 K^T(B + \Delta B)^T Q(B + \Delta B)K + \alpha_1^2 C_2^T L^T QLC_2$$
$$+(C_1 + \Delta C_1)^T(C_1 + \Delta C_1) - P + \tau_1 G^T G$$

$$\breve{\Omega} = \bar{\beta}^2 K^T(B + \Delta B)^T P(B + \Delta B)K$$
$$+\beta_1^2 K^T(B + \Delta B)^T P(B + \Delta B)K$$
$$+\beta_1^2 K^T(B + \Delta B)^T Q(B + \Delta B)K$$
$$+((A + \Delta A) - \bar{\alpha}LC_2)^T Q((A + \Delta A) - \bar{\alpha}LC_2) - Q$$

$$+\tau_2 G^T G$$

$$\hat{\Omega} = D^T P D + (D - LD_2)^T Q(D - LD_2) + D^T D_1 - \gamma^2 I$$

$$\Omega_2 = \bar{\beta} K^T (B + \Delta B)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)$$

$$-\beta_1^2 K^T (B + \Delta B)^T P(B + \Delta B)K$$

$$-\beta_1^2 K^T (B + \Delta B)^T Q(B + \Delta B)K$$

$$\Omega_3 = \bar{\beta} D^T P(B + \Delta B)K$$

$$+(D - LD_2)^T Q((A + \Delta A) - \bar{\alpha} LC_2)$$

$$\Omega_4 = D^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K) + D_1^T (C_1 + \Delta C_1)$$

$$\Omega_5 = P(A + \Delta A) - \bar{\beta} P(B + \Delta B)K$$

$$\Omega_6 = Q((A + \Delta A) - \bar{\alpha} LC_2)$$

Or it can be in the equivlent form of a MI as shown below

$$\Omega - \tau_1 \Omega_1 - \tau_2 \Omega_2$$
$$= \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & 0 \\ 0 & -Q + \tau_2 G^T G^T & * & * & 0 \\ 0 & 0 & -\gamma^2 I & * & 0 \\ 0 & 0 & 0 & -\tau_1 I & 0 \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$$

$$+\begin{bmatrix} \tilde{\Omega} & * & * & * & * \\ \Omega_2 & \breve{\Omega} & * & * & * \\ \Omega_3 & \Omega_4 & \hat{\Omega} & * & * \\ \Omega_5 & \bar{\beta}P(B+\Delta B)K & PD & P & * \\ 0 & \Omega_6 & Q(D-LD_2) & 0 & Q \end{bmatrix} < 0$$

where

$$\tilde{\Omega} = ((A+\Delta A) - \bar{\beta}(B+\Delta B)K)^T P((A+\Delta A)$$

$$-\bar{\beta}(B+\Delta B)K) + \beta_1^2 K^T (B+\Delta B)^T P(B+\Delta B)K$$

$$+\beta_1^2 K^T (B+\Delta B)^T Q(B+\Delta B)K$$

$$+\alpha_1^2 C_2^T L^T Q L C_2 + C_1^T (C_1 - \Delta C_1)$$

$$\breve{\Omega} = \bar{\beta}^2 K^T (B+\Delta B)^T P(B+\Delta B)K$$

$$+\beta_1^2 K^T (B+\Delta B)^T P(B+\Delta B)K$$

$$+\beta_1^2 K^T (B+\Delta B)^T Q(B+\Delta B)K$$

$$+((A+\Delta A) - \bar{\alpha}LC_2)^T Q((A+\Delta A) - \bar{\alpha}LC_2)$$

$$\hat{\Omega} = D^T PD + (D-LD_2)^T Q(D-LD_2) + D^T D_1$$

$$\Omega_2 = \bar{\beta}K^T (B+\Delta B)^T P((A+\Delta A) - \bar{\beta}(B+\Delta B)K)$$

$$-\beta_1^2 K^T (B+\Delta B)^T P(B+\Delta B)K$$

$$-\beta_1^2 K^T (B+\Delta B)^T Q(B+\Delta B)K$$

$$\Omega_3 = D^T P((A+\Delta A) - \bar{\beta}(B+\Delta B)K) + D_1^T (C_1 + \Delta C_1)$$

$$\Omega_4 = \bar{\beta} D^T P(B + \Delta B)K$$

$$+(D - LD_2)^T Q((A + \Delta A) - \bar{\alpha} LC_2)$$

$$\Omega_5 = P(A + \Delta A) - \bar{\beta} P(B + \Delta B)K$$

$$\Omega_6 = Q((A + \Delta A) - \bar{\alpha} LC_2) = Q\,(A + A\Delta - L\alpha C_2).$$

So it can be rewritten in the following form.

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2$$
$$= \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & 0 \\ 0 & -Q + \tau_2 G^T G^T & * & * & 0 \\ 0 & 0 & -\gamma^2 I & * & 0 \\ 0 & 0 & 0 & -\tau_1 I & 0 \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix}$$
$$+ \begin{bmatrix} \Phi_1 & \Phi_2 \end{bmatrix}$$
$$\times \begin{bmatrix} P^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & P^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & Q^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$\times \begin{bmatrix} P((A+\Delta A)-\bar{\beta}(B+\Delta B)K) & \bar{\beta}P(B+\Delta B)K & PD & P & 0 \\ \beta_1 P(B+\Delta B)K & -\beta_1 P(B+\Delta B)K & 0 & 0 & 0 \\ \beta_1 Q(B+\Delta B)K & -\beta_1 Q(B+\Delta B)K & 0 & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 \\ 0 & Q((A+\Delta A)-\bar{\alpha}LC_2) & Q(D-LD_2) & 0 & Q \\ C_1 & 0 & D_1 & 0 & 0 \end{bmatrix} <$$

0

where

$$\Phi_1 = \begin{bmatrix} ((A+\Delta A)-\bar{\beta}(B+\Delta B)K)^T P & \beta_1 K^T (B+\Delta B)^T P \\ \bar{\beta}K^T(B+\Delta B)^T P & -\beta_1 K^T(B+\Delta B)^T P \\ D^T P & 0 \\ P & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} \beta_1 K^T(B+\Delta B)^T Q & \alpha_1 C_2^T L^T Q & 0 & C_1^T \\ -\beta_1 K^T(B+\Delta B)^T Q & 0 & ((A+\Delta A)-\bar{\alpha}LC_2)^T Q & 0 \\ 0 & 0 & (D-LD_2)^T Q & D_1^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & Q & 0 \end{bmatrix}$$

Also, $\Omega - \tau_1\Omega_1 - \tau_2\Omega_2$ can be rewritten in the other form.

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2 = \begin{bmatrix} \Gamma_{11} & \Gamma_{12}^T \\ \Gamma_{12} & \Gamma_{22} \end{bmatrix} + \begin{bmatrix} 0 & \tilde{\Gamma}^T \\ \tilde{\Gamma} & 0 \end{bmatrix}$$

where

$$\Gamma_{11} = \begin{bmatrix} -P+\tau_1 G^T G^T & * & * & * & 0 \\ 0 & -Q+\tau_2 G^T G^T & * & * & 0 \\ 0 & 0 & -\gamma^2 I & * & 0 \\ 0 & 0 & 0 & -\tau_1 I & 0 \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} -P & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} PA-\bar{\beta}PBK & \bar{\beta}PBK & PD & P & 0 \\ \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 \\ \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 \\ 0 & QA-\bar{\alpha}QLC_2 & Q(D-LD_2) & 0 & Q \\ C_1 & 0 & D_1 & 0 & 0 \end{bmatrix} \text{snd}$$

$$\tilde{\Gamma} = \begin{bmatrix} P\Delta A-\bar{\beta}P\Delta BK & \bar{\beta}P(\Delta B)K & 0 & 0 & 0 \\ \beta_1 P(\Delta B)K & -\beta_1 P(\Delta B)K & 0 & 0 & 0 \\ \beta_1 Q(\Delta B)K & -\beta_1 Q(\Delta B)K & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & Q(\Delta A) & 0 & 0 & 0 \\ (\Delta C_1) & 0 & 0 & 0 & 0 \end{bmatrix}$$

Based on (4.4) , Lemma (4.20) and Lemma (4.21) it can be rewritten in the following form:

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2 = \begin{bmatrix} \Gamma_{11} & \Sigma^T \\ \Sigma & \Gamma_{22} \end{bmatrix} + \left[\tilde{\Theta}\right] \Delta \left[\breve{\Theta}\right] + \left[\breve{\Theta}\right]^T \Delta^T \left[\tilde{\Theta}\right]^T$$

where

$$\Gamma_{11} = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & 0 \\ 0 & -Q + \tau_2 G^T G^T & * & * & 0 \\ 0 & 0 & -\gamma^2 I & * & 0 \\ 0 & 0 & 0 & -\tau_1 I & 0 \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} -P & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} PA - \bar{\beta}PBK & \bar{\beta}PBK & PD & P & 0 \\ \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 \\ \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 \\ 0 & QA - \bar{\alpha}QLC_2 & Q(D - LD_2) & 0 & Q \\ C_1 & 0 & D_1 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Theta} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (PM_c) & -\bar{\beta}\,(PM_c) & 0 & 0 \\ 0 & \beta_1\,(PM_c) & 0 & 0 \\ 0 & \beta_1\,(QM_c) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (QM_c) \\ 0 & 0 & N_3 & 0 \end{bmatrix},$$

$$\breve{\Theta} = \begin{bmatrix} (N_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (N_2K) & (N_2K) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_c^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$\Delta = \begin{bmatrix} \Delta_k & 0 & 0 & 0 \\ 0 & \Delta_k & 0 & 0 \\ 0 & 0 & \Delta_k & 0 \\ 0 & 0 & 0 & \Delta_k \end{bmatrix}$$

from the constraint (4.5), $(\Delta_k \Delta_k^T) < I$, can be hold for the condition that there is $\epsilon > 0$ such that

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2 = \begin{bmatrix} \tilde{\Pi} & \Psi^T \\ \Psi & \breve{\Pi} \end{bmatrix} < 0$$

where

$$\tilde{\Pi} = \begin{bmatrix} -P+\tau_1 G^T G^T & * & * & * & 0 \\ 0 & -Q+\tau_2 G^T G^T & * & * & 0 \\ 0 & 0 & -\gamma^2 I & * & 0 \\ 0 & 0 & 0 & -\tau_1 I & 0 \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\breve{\Pi} = \begin{bmatrix} \hat{\Pi} & \Sigma^T \\ \Sigma & \check{\Pi} \end{bmatrix}$$

$$\hat{\Pi} = \begin{bmatrix} -P & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\check{\Pi} = \begin{bmatrix} -\epsilon I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\epsilon I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\epsilon I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\epsilon I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{and}$$

$$\Psi = \begin{bmatrix} PA - \bar{\beta}PBK & \bar{\beta}PBK & PD & P & 0 \\ \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 \\ \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 \\ \alpha_1 QLC_2 & 0 & 0 & 0 & 0 \\ 0 & QA - \bar{\alpha}QLC_2 & Q(D - LD_2) & 0 & Q \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon\,(N_1) & 0 & 0 & 0 & 0 \\ \epsilon\,(N_2K) & \epsilon\,(N_2K) & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 \end{bmatrix}.$$

This will give the same BMI as shown in (4.32).Then by multiplying the left side of the above MI by the transpose of the $daig\{I, I, I, I, P^{-1}, \ , Q^{-1}, Q^{-1}, Q^{-1}, I, I, I, I, I, I, I, I\}$

and the right side without transpose. It has resulted in the following MI .

$$\Omega - \tau_1\Omega_1 - \tau_2\Omega_2 = \begin{bmatrix} \Xi_{11} & \Upsilon^T \\ \Upsilon & \Xi_{22} \end{bmatrix} < 0 \tag{4.39}$$

where

$$\Xi_{11} = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Xi_{22} = \begin{bmatrix} \digamma_{11} & \Psi^T \\ \Psi & \digamma_{22} \end{bmatrix}$$

$$\digamma_{11} = \begin{bmatrix} -P^{-1} & 0 & 0 & * & * & * \\ 0 & -P^{-1} & 0 & * & * & * \\ 0 & 0 & -Q^{-1} & * & * & * \\ 0 & 0 & 0 & -Q^{-1} & * & * \\ 0 & 0 & 0 & 0 & -Q^{-1} & * \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\digamma_{22} = \begin{bmatrix} -\epsilon I & * & * & * & * & * & * & * \\ 0 & -\epsilon I & 0 & * & * & * & * & * \\ 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & 0 & -\epsilon I & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Psi = \begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Upsilon = \begin{bmatrix} A-\bar{\beta}BK & \bar{\beta}BK & D & I & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \\ 0 & A-\bar{\alpha}LC_2 & (D-LD_2) & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon\,(N_1) & 0 & 0 & 0 & 0 \\ \epsilon\,(N_2K) & \epsilon\,(N_2K) & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 \end{bmatrix}.$$

So by using schur complement, the inequailty in (4.32) is equivalent to (4.39). At the end, the previous steps have ended in the following inequality.

$$\zeta_k^T \Omega \zeta_k^T < 0 \tag{4.40}$$

Thus, it can be concluded from (4.33) and (4.40) that

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} < 0 \tag{4.41}$$

Now, by summing (4.41) from 0 to ∞ with respect to k which will yield with the following.

$$\sum_{k=0}^{\infty}\mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty}\mathbb{E}\{w_k^T w_k\} - \mathbb{E}\{V_0\} - \mathbb{E}\{V_\infty\} \tag{4.42}$$

Where the closed-loop networked nonlinear system (4.18) is exponentially mean square stable and $\eta_0 = 0$, it can be ended to the conclusion.

$$\sum_{k=0}^{\infty}\mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty}\mathbb{E}\{w_k^T w_k\} \tag{4.43}$$

Finally, the specified H_∞ performance constraints (4.19) have been achieved. This ends the proof. □

Now after finding the basic model, it still needs a special logic solver to find the controller gain and the observer gain because of its complexity.

4.3.2 Finding Suitable Approach

In order to solve the MI in (4.39) and to design the controller and the observer gains easily, it is needed to derive suitable LMI approaches to make this MI solvable in the environment of a linear logic solver.

In order to find an LMI approach and to design the controller gains without reservations, two suggestions have been created as the following.

1- Removing the ϵ either by suggesting $\epsilon = 1$.or making $\bar{K} = \epsilon K$

2-Changing P^{-1}, Q^{-1} to an equivalent matrix .

The first approach for P^{-1}, Q^{-1}can be made from the matrix (4.39), which can be held if there is $\Omega_3 > 0$ such that

$\hat{\Omega} - \Omega_3 < 0$

where $\Omega_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \tilde{N} & 0 \\ 0 & 0 & 0 \end{bmatrix}$,$\tilde{N} = \begin{bmatrix} N_{11} & 0 & 0 & 0 & 0 \\ 0 & N_{11} & 0 & 0 & 0 \\ 0 & 0 & N_{22} & 0 & 0 \\ 0 & 0 & 0 & N_{22} & 0 \\ 0 & 0 & 0 & 0 & N_{22} \end{bmatrix}$, 0 is zero matrix with proper dimensions and $\hat{\Omega} = \Omega - \tau_1\Omega_1 - \tau_2\Omega_2$.

N_{11}, N_{22} are matrices with the same dimensions of P, Q respectively such that

$N_{11} = (\tilde{N}_1 - P^{-1})$

$N_{22} = (\tilde{N}_2 - Q^{-1})$

Where $\tilde{N}_1$and $\tilde{N}_2$ are positive-definite matrices

Therefore, $\hat{\Omega} - \Omega_3$ can be shaped in the following LMI.

$$\hat{\Omega} - \Omega_3 = \begin{bmatrix} \Xi_{11} & \Upsilon_1^T \\ \Upsilon_1 & \Xi_{22} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & \tilde{N} & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0$$

where

$$\tilde{N} = \begin{bmatrix} N_{11} & 0 & 0 & 0 & 0 \\ 0 & N_{11} & 0 & 0 & 0 \\ 0 & 0 & N_{22} & 0 & 0 \\ 0 & 0 & 0 & N_{22} & 0 \\ 0 & 0 & 0 & 0 & N_{22} \end{bmatrix},$$

$$\Xi_{11} = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\Xi_{22} = \begin{bmatrix} \mathcal{F}_{11} & \Psi_1^T \\ \Psi_1 & \mathcal{F}_{22} \end{bmatrix}$$

$$\mathcal{F}_{11} = \begin{bmatrix} -P^{-1} & 0 & 0 & * & * & * \\ 0 & -P^{-1} & 0 & * & * & * \\ 0 & 0 & -Q^{-1} & * & * & * \\ 0 & 0 & 0 & -Q^{-1} & * & * \\ 0 & 0 & 0 & 0 & -Q^{-1} & * \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$F_{22}=\begin{bmatrix} -\epsilon I & * & * & * & * & * & * & * \\ 0 & -\epsilon I & 0 & * & * & * & * & * \\ 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & 0 & -\epsilon I & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Psi_1=\begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Upsilon_1 = \begin{bmatrix} A-\bar{\beta}BK & \bar{\beta}BK & D & I & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \\ 0 & A-\bar{\alpha}LC_2 & (D-LD_2) & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon(N_1) & 0 & 0 & 0 & 0 \\ \epsilon(N_2K) & \epsilon(N_2K) & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, it can be rewritten as the first approach with making $\epsilon = 1$ as shown in the following LMI.

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Upsilon_1^T \\ \Upsilon_1 & \tilde{\Xi}_{22} \end{bmatrix} < 0 \tag{4.44}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P+\tau_1 G^T G & * & * & * & * \\ 0 & -Q+\tau_2 G^T G & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\tilde{\Xi}_{22} = \begin{bmatrix} \tilde{\mathcal{F}}_{11} & \Psi_1^T \\ \Psi_1 & \tilde{\mathcal{F}}_{22} \end{bmatrix},$$

$$\tilde{\mathcal{F}}_{11} = \begin{bmatrix} -\tilde{N}_1 & 0 & 0 & * & * & * \\ 0 & -\tilde{N}_1 & 0 & * & * & * \\ 0 & 0 & -\tilde{N}_2 & * & * & * \\ 0 & 0 & 0 & -\tilde{N}_2 & * & * \\ 0 & 0 & 0 & 0 & -\tilde{N}_2 & * \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{\mathcal{F}}_{22} = \begin{bmatrix} -I & * & * & * & * & * & * & * \\ 0 & -I & 0 & * & * & * & * & * \\ 0 & 0 & -I & * & * & * & * & * \\ 0 & 0 & 0 & -I & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Upsilon_1 = \begin{bmatrix} A-\bar{\beta}BK & \bar{\beta}BK & D & I & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \\ 0 & A-\bar{\alpha}LC_2 & (D-LD_2) & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ (N_1) & 0 & 0 & 0 & 0 \\ (N_2K) & (N_2K) & 0 & 0 & 0 \\ M_c^T & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 \end{bmatrix}.$$

The approach can also be rewritten by considering $\bar{K} = \epsilon K$ and LMI will be in the following shape.

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Upsilon_1^T \\ \Upsilon_1 & \tilde{\Xi}_{22} \end{bmatrix} < 0 \tag{4.45}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P + \tau_1 G^T G^T & * & * & * & * \\ 0 & -Q + \tau_2 G^T G^T & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\tilde{\Xi}_{22} = \begin{bmatrix} \tilde{F}_{11} & \Psi_1^T \\ \Psi_1 & \tilde{F}_{22} \end{bmatrix},$$

$$\tilde{F}_{11} = \begin{bmatrix} -\tilde{N}_1 & 0 & 0 & * & * & * \\ 0 & -\tilde{N}_1 & 0 & * & * & * \\ 0 & 0 & -\tilde{N}_2 & * & * & * \\ 0 & 0 & 0 & -\tilde{N}_2 & * & * \\ 0 & 0 & 0 & 0 & -\tilde{N}_2 & * \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{F}_{22} = \begin{bmatrix} -\epsilon I & * & * & * & * & * & * & * \\ 0 & -\epsilon I & 0 & * & * & * & * & * \\ 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & 0 & -\epsilon I & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{and}$$

$$\Upsilon_1 = \begin{bmatrix} A-\bar{\beta}BK & \bar{\beta}BK & D & I & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \\ 0 & A-\bar{\alpha}LC_2 & (D-LD_2) & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon\,(N_1) & 0 & 0 & 0 & 0 \\ \left(N_2\bar{K}\right) & \left(N_2\bar{K}\right) & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 \end{bmatrix}.$$

It is noted in this format that there are unknown parameters matrices more than the symmetric matrices P and Q which are $\tilde{N}_1$and $\tilde{N}_2$.

Even though it can be solved by the LMI solver but to reduce the number of the unknown paramete matrices, the following LMIs have been suggested.

In order to solve this LMI and to design the controller easily the following modification is suggested by [42].

$-P^{-1}$ and $-Q^{-1}$are replaced by $P - 2I$ and $Q - 2I$ respectively.

Therefore, the third LMI approach can be written by considering $\epsilon = 1$ as shown in the following form.

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Upsilon_3^T \\ \Upsilon_3 & \breve{\Xi}_{22} \end{bmatrix} < 0 \tag{4.46}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P + \tau_1 G^T G & * & * & * & * \\ 0 & -Q + \tau_2 G^T G & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\breve{\Xi}_{22} = \begin{bmatrix} \breve{F}_{11} & \Psi_1^T \\ \Psi_1 & \tilde{F}_{22} \end{bmatrix},$$

$$\breve{F}_{11} = \begin{bmatrix} P-2I & 0 & 0 & * & * & * \\ 0 & P-2I & 0 & * & * & * \\ 0 & 0 & Q-2I & * & * & * \\ 0 & 0 & 0 & Q-2I & * & * \\ 0 & 0 & 0 & 0 & Q-2I & * \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{F}_{22} = \begin{bmatrix} -I & * & * & * & * & * & * & * \\ 0 & -I & 0 & * & * & * & * & * \\ 0 & 0 & -I & * & * & * & * & * \\ 0 & 0 & 0 & -I & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Upsilon_1 = \begin{bmatrix} A-\bar{\beta}BK & \bar{\beta}BK & D & I & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \\ 0 & A-\bar{\alpha}LC_2 & (D-LD_2) & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ (N_1) & 0 & 0 & 0 & 0 \\ (N_2K) & (N_2K) & 0 & 0 & 0 \\ M_c^T & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 \end{bmatrix}.$$

The same LMI can be written in a fourth LMI form by suggesting $\bar{K} = \epsilon K$ as shown in the following.

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Upsilon_3^T \\ \Upsilon_3 & \breve{\Xi}_{22} \end{bmatrix} < 0 \tag{4.47}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P + \tau_1 G^T G & * & * & * & * \\ 0 & -Q + \tau_2 G^T G & * & * & * \\ 0 & 0 & -\gamma I & * & * \\ 0 & 0 & 0 & -\tau_1 I & * \\ 0 & 0 & 0 & 0 & -\tau_2 I \end{bmatrix},$$

$$\breve{\Xi}_{22} = \begin{bmatrix} \breve{\mathcal{F}}_{11} & \Psi_1^T \\ \Psi_1 & \tilde{\mathcal{F}}_{22} \end{bmatrix},$$

$$\breve{\mathcal{F}}_{11} = \begin{bmatrix} P - 2I & 0 & 0 & * & * & * \\ 0 & P - 2I & 0 & * & * & * \\ 0 & 0 & Q - 2I & * & * & * \\ 0 & 0 & 0 & Q - 2I & * & * \\ 0 & 0 & 0 & 0 & Q - 2I & * \\ 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{\mathcal{F}}_{22} = \begin{bmatrix} -\epsilon I & * & * & * & * & * & * & * \\ 0 & -\epsilon I & 0 & * & * & * & * & * \\ 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & 0 & -\epsilon I & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} (PM_c)^T & 0 & 0 & 0 & 0 & 0 \\ -\bar{\beta}\,(PM_c)^T & \beta_1\,(PM_c)^T & \beta_1\,(QM_c)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3^T \\ 0 & 0 & 0 & 0 & (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Upsilon_1 = \begin{bmatrix} A-\bar{\beta}BK & \bar{\beta}BK & D & I & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 \\ \alpha_1 LC_2 & 0 & 0 & 0 & 0 \\ 0 & A-\bar{\alpha}LC_2 & (D-LD_2) & 0 & I \\ C_1 & 0 & D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \epsilon\,(N_1) & 0 & 0 & 0 & 0 \\ \left(N_2\bar{K}\right) & \left(N_2\bar{K}\right) & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 \end{bmatrix}.$$

As a result to that, in order to establish the observer-based controller (4.10) and (4.11) for the networked nonlinear system (4.1) under the H_∞ peformance constraint (4.19) with minum γ, it is suposed to consider the following optimization problem

$$\min_{P>0,Q>0\tilde{N}_1>0,\tilde{N}_2>0,\tau_1>0,\tau_2>0,} \gamma \tag{4.48}$$

subject to inequality given in (4.44),(4.45),(4.46)or(4.47)

Finally, the following two corollary have been concluded.

Corollary 4.3.1. *Given communication channel parametes* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$, *If the optimization problem (4.48) is feasible, the observer-based control law (4.11) with controller parameters K and L that can be found by LMI described in (4.44) will exponentially stabilize the uncertain networked nonlinear system (4.1) with minimum* H_∞ *Performance bound* $\gamma_{\min}$.

Remark 4.3.1. *Corollary* 4.3.1*can aslo be rewritten in the second suggestion which is shown in (4.45) .*

Corollary 4.3.2. *Given communication channel parametes* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$, *If the optimization problem (4.48) is feasible, the observer-based control law (4.11) with controller parameters K and L can be found by LMI described in (4.46) or(4.47) will* *exponentially stabilize the uncertain networked nonlinear system (4.1) with minimum* H_∞ *Performance bound* $\gamma_{\min}$.

Remark 4.3.2. *Corollary* 4.3.2*can aslo be rewritten in the second suggestion which is shown in (4.45) .*

4.4 Numerical Example

In this section, a matlab code has been made for the last created LMIs that were shown in (4.44), (4.45), (4.46) and (4.46) . Considering a system described by (4.1) and the measurement equation(4.6) . The following given parameters have been applied in this system.

$$A = \begin{bmatrix} 0.8226 & -0.633 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 23.738 & 20.287 & 0 \end{bmatrix},$$

$$D_1 = 0.1\;, \;\; D_2 = 0.2,$$

$$f(k, x_k) = \begin{bmatrix} 0.01 \sin\; x_k^1 \\ 0.01 \sin\; x_k^2 \\ 0.01 \sin\; x_k^3 \end{bmatrix} \quad x_k = \begin{bmatrix} x_k^1 \\ x_k^2 \\ x_k^3 \end{bmatrix}, \; G = [\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \end{bmatrix}, N_2 = \begin{bmatrix} 0.2 \\ 0 \\ 0 \end{bmatrix}, N_3 = \begin{bmatrix} 0.3 & 0 & 0 \end{bmatrix} \text{ and } M_c = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1 & 0 & 0 \end{bmatrix}$$

The initial conditions of the networked nonlinear system (4.1) have been assumed as $x_0 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix}$ $\hat{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, and the disturbance input has been supposed to be $\omega_k = 1/k^2$.

The aim is to design the controller (4.11) for system (4.1), such that the H_∞ performance index is minimized.

The simulation has been made for the two different suggestions regarding ϵ either by suggestion $\epsilon = 1$ or by adding another unknown matrix $\bar{K}$ that has been defined as $\bar{K} = \epsilon K$ .Each suggestion was applied for the other two approaches . The results have been summarized in the table (4.1).

It has been noticed that these approaches are near each other in the behaviour of the stability. In genral it has been noticed that the minimization is the best for approach (4.44) but it has more reservation. The minimization performance in the fourth approach (4.47) has the higher value but with quick settling in stability. At the extremes limits values of the packet losses, the behaviour of the stability has been kept with little increase in the values of the minimization factor and in the settling time but in general they have been near to each other. The performance factor has kept balanced in all the four LMI approaches for different

First LMI approach (at $\epsilon = 1$ with $\tilde{N}$)					
Figure#	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
4.1	0.98	0.9	$\begin{bmatrix} 0.3452 & -0.2454 & 0.0020 \end{bmatrix}$	$\begin{bmatrix} 0.007 5 & 0.012 4 & 0.020 7 \end{bmatrix}^T$	1.3201
4.2	0.5	0.4	$\begin{bmatrix} 0.2914 & -0.1862 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0071 & 0.012 2 & 0.020 9 \end{bmatrix}^T$	1.8321
4.3	0.02	0.1	$\begin{bmatrix} 0.1788 & -0.1376 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0120 & 0.0121 & 0.0228 \end{bmatrix}^T$	1.8428
Second LMI approach (at $\epsilon K = \bar{K}$ with $\tilde{N}$)					
Figure#	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
4.7	0.98	0.9	$\begin{bmatrix} 0.3739 & -0.2877 & 0.0020 \end{bmatrix}$	$\begin{bmatrix} 0.007 & 0.012 2 & 0.020 8 \end{bmatrix}^T$	3.6100
4.8	0.5	0.4	$\begin{bmatrix} 0.2571 & -0.1978 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0071 & 0.012 2 & 0.020 9 \end{bmatrix}^T$	3.6640,
4.9	0.02	0.1	$\begin{bmatrix} 0.2165 & -0.1666 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.012 & 0.0121 & 0.0228 \end{bmatrix}^T$	3.7061
Third LMI approach (at $\epsilon = 1$ with $P - 2I$ &$Q - 2I$)					
Figure#	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
4.4	0.98	0.9	$\begin{bmatrix} 0.3594 & -0.2766 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0070 & 0.0122 & 0.020 8 \end{bmatrix}^T$	2.0937
4.5	0.5	0.4	$\begin{bmatrix} 0.2914 & -0.1862 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0071 & 0.012 2 & 0.0209 \end{bmatrix}^T$	2.3830
4.6	0.02	0.1	$\begin{bmatrix} 0.1788 & -0.1376 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.012 & 0.012 1 & 0.0228 \end{bmatrix}^T$	2.3905
Fourth LMI approach (at $\epsilon K = \bar{K}$ with $P - 2I$ &$Q - 2I$)					
Figure#	$\bar{\alpha}$	$\bar{\beta}$	K	L	$\gamma_{\min}$
4.10	0.98	0.9	$\begin{bmatrix} 0.3739 & -0.2877 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.007 & 0.012 2 & 0.020 8 \end{bmatrix}^T$	5.1379
4.11	0.5	0.4	$\begin{bmatrix} 0.2571 & -0.1978 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0071 & 0.0122 & 0.020 9 \end{bmatrix}^T$	5.1830
4.12	0.02	0.1	$\begin{bmatrix} 0.2165 & -0.1666 & 0.0000 \end{bmatrix}$	$\begin{bmatrix} 0.0012 & 0.0121 & 0.0228 \end{bmatrix}^T$	5.2228

TABLE 4.1. Summary for four LMI approaches

values of the packet losses. It appears that approach (136) has the best performance even though it has higher reservation more than LMI in (4.44) but it has the best minimization factor of H_∞ performance .

4.5 Conclusion

At the end of this chapter it can be concluded that an observer-based H_∞ controller has been designed for a class of uncertain NNCS. The packet loss change has been described as property of Bernoulli distribution and the distribution of the packet loss from the sensor to the controller has been considered different from the controller to the actuator. Certain conditions have succeeded to make the system work in an exponential mean square stabllity. It is based on the work of[4],[5] and[50]to create four LMI approaches. These four LMIs have succeeded to achieve the H_∞ minimization performance.. The numerical example has explained the effects of these LMIs on the system stability with more options..

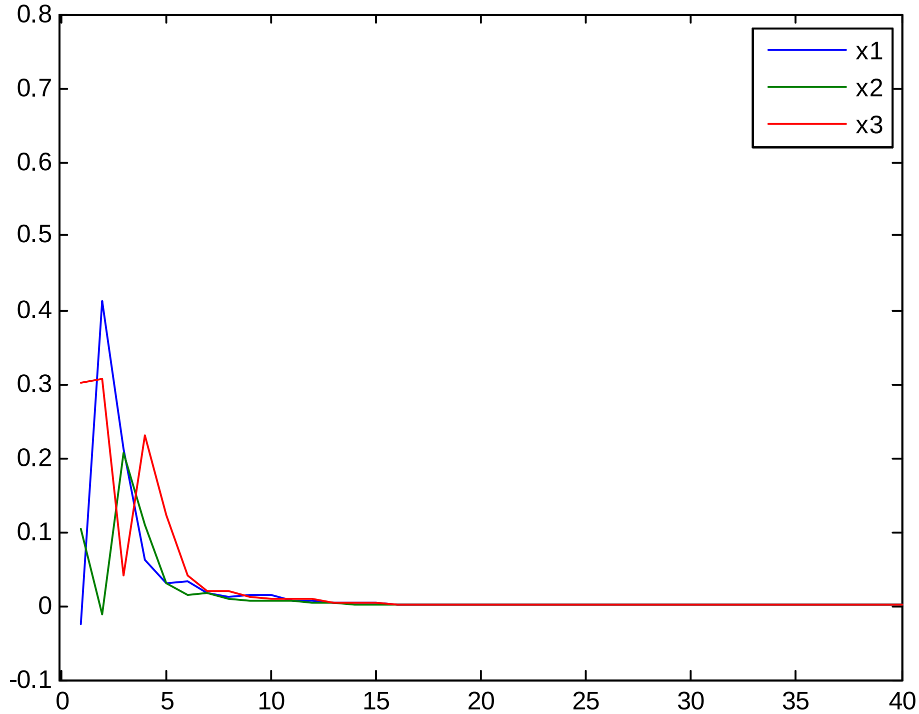

FIGURE 4.1. Observer-based H_∞ controller design by approach (4.44) with $\bar{\alpha} = 0.98$ and $\bar{\beta} = 0.9$

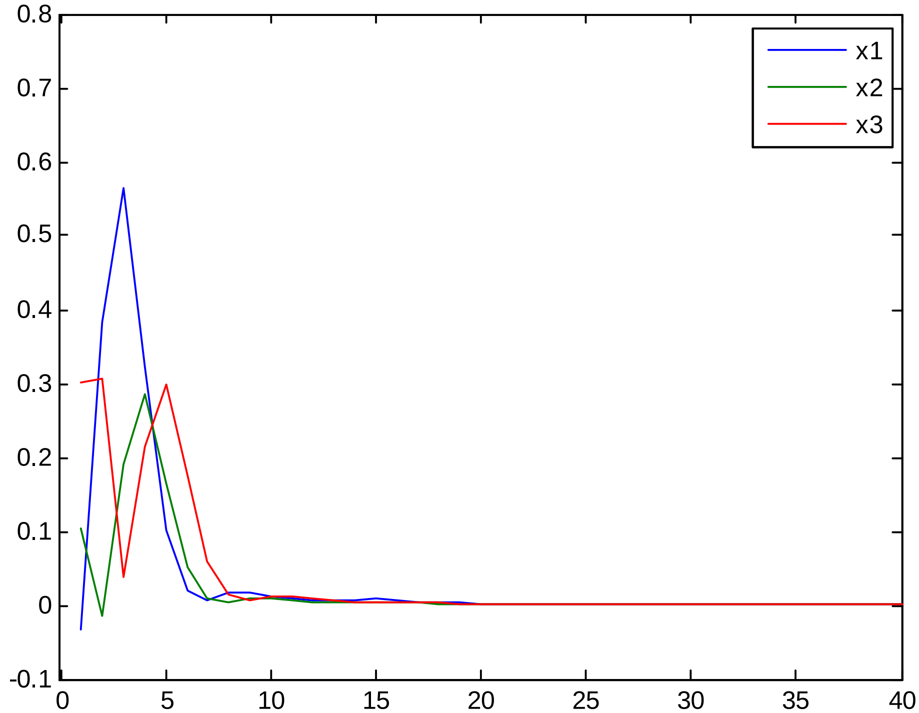

FIGURE 4.2. Observer-based H_∞ controller design by approach (4.44) with $\bar{\alpha} = 0.5$ and $\bar{\beta} = 0.4$

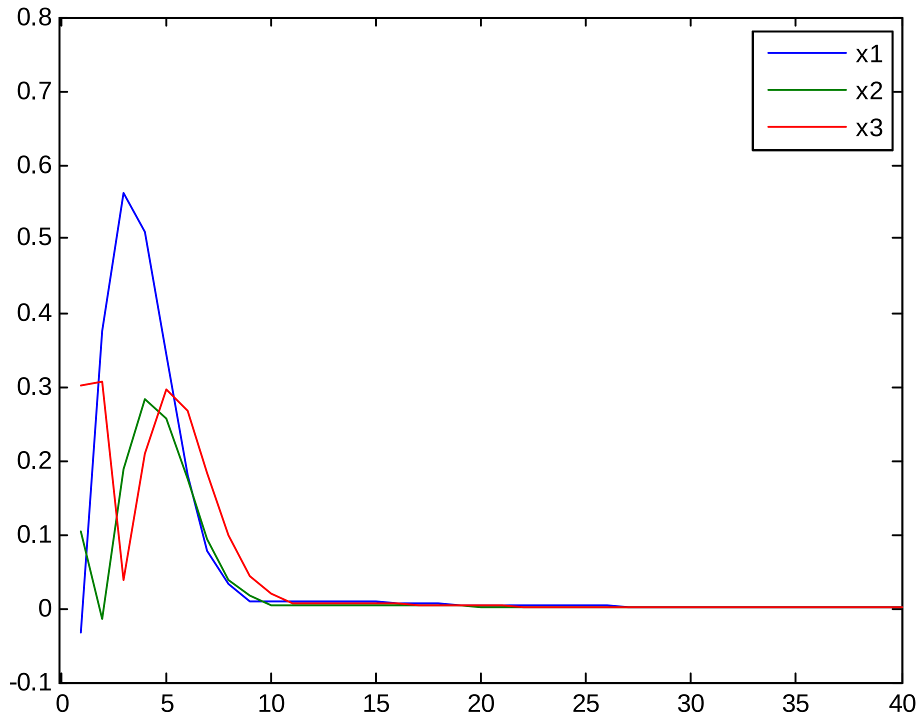

FIGURE 4.3. Observer-based H_∞ controller design by approach (4.44) with $\bar{\alpha} = 0.02$ and $\bar{\beta} = 0.1$

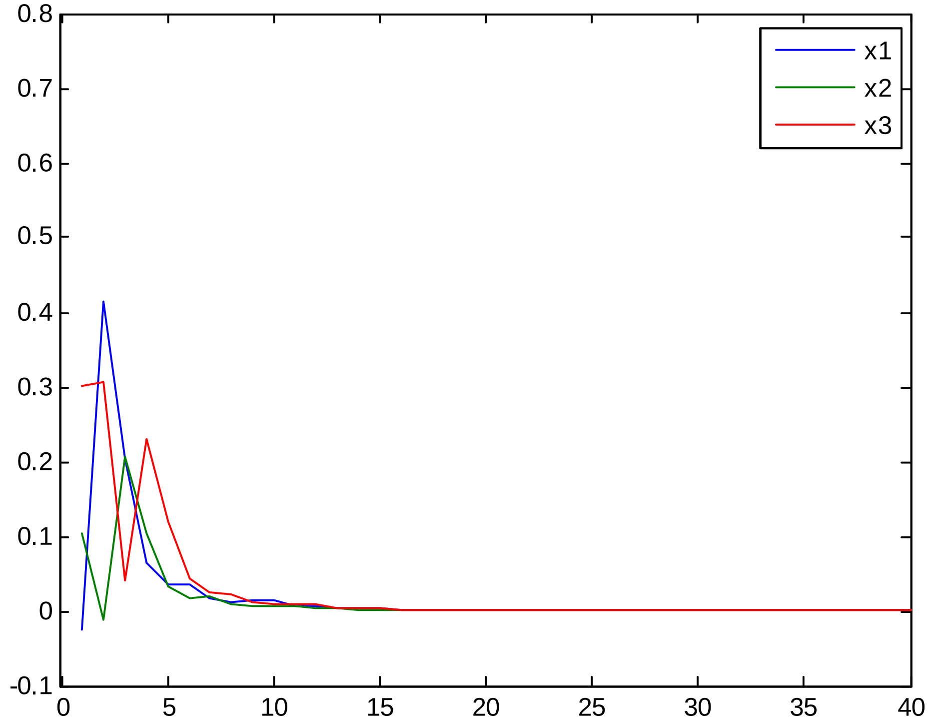

FIGURE 4.4. Observer-based H_∞ controller design by approach (4.46) with $\bar{\alpha} = 0.98$ and $\bar{\beta} = 0.9$

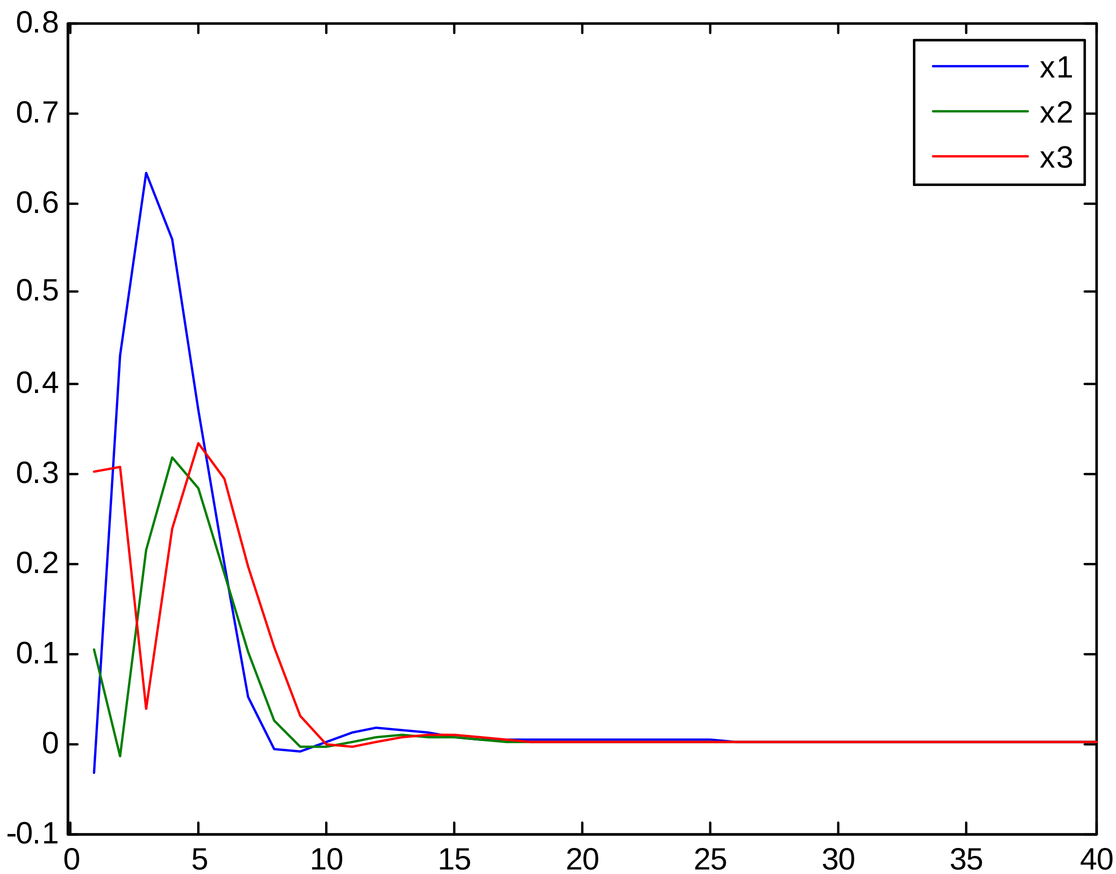

FIGURE 4.5. Observer-based H_∞ controller design by approach (4.46) with $\bar{\alpha} = 0.5$ and $\bar{\beta} = 0.4$

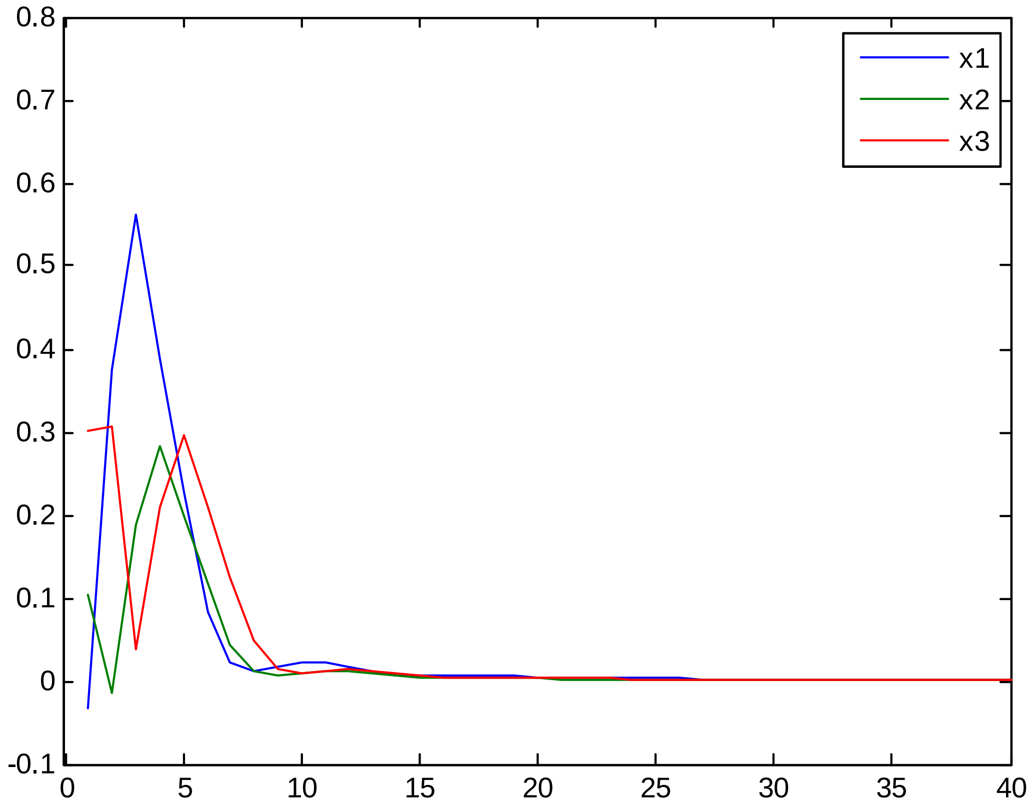

FIGURE 4.6. Observer-based H_∞ controller design by approach (4.46) with $\bar{\alpha} = 0.02$ and $\bar{\beta} = 0.1$

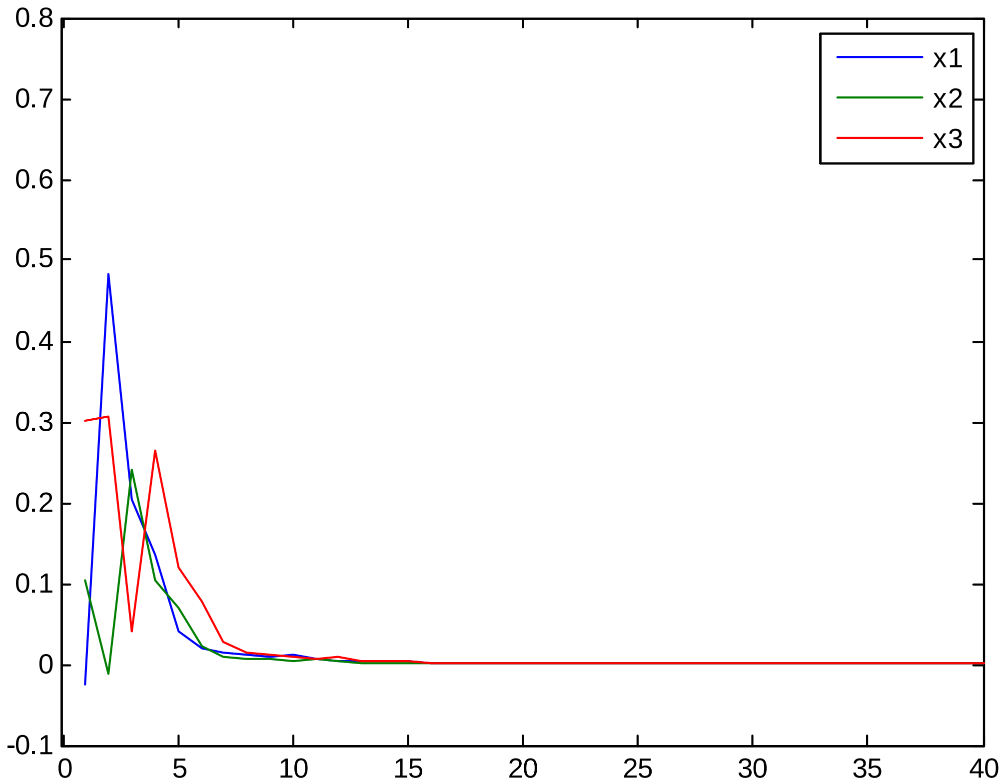

FIGURE 4.7. Observer-based H_∞ controller design by approach (4.45) with $\bar{\alpha} = 0.98$ and $\bar{\beta} = 0.9$

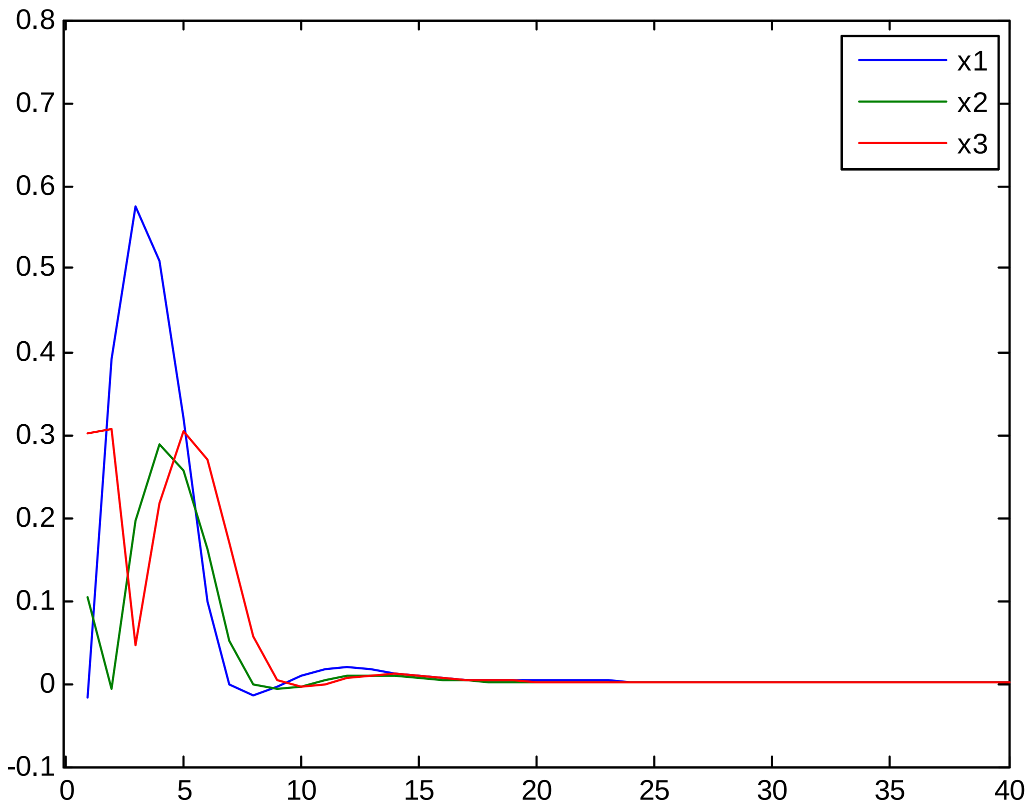

FIGURE 4.8. Observer-based H_∞ controller design by approach (4.45) with $\bar{\alpha} = 0.5$ and $\bar{\beta} = 0.4$

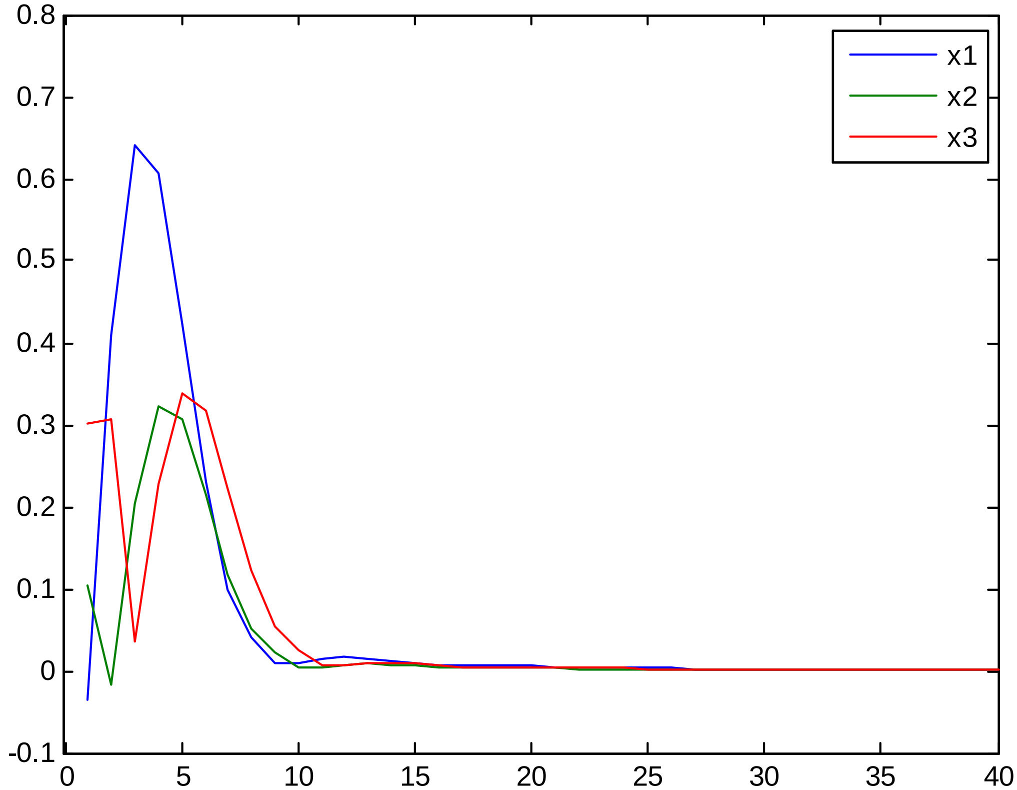

FIGURE 4.9. Observer-based H_∞ controller design by approach (4.45) with $\bar{\alpha} = 0.02$ and $\bar{\beta} = 0.1$

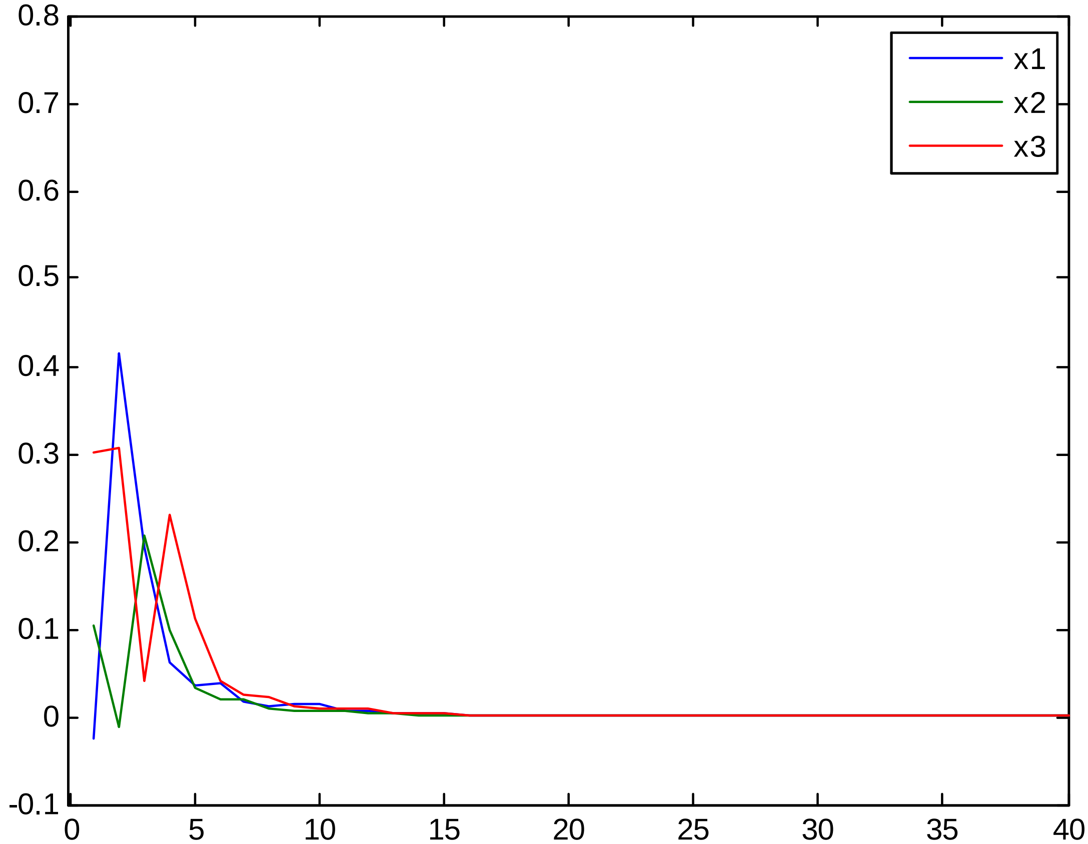

FIGURE 4.10. Observer-based H_∞ controller design by approach (4.47) with $\bar{\alpha} = 0.98$ and $\bar{\beta} = 0.9$

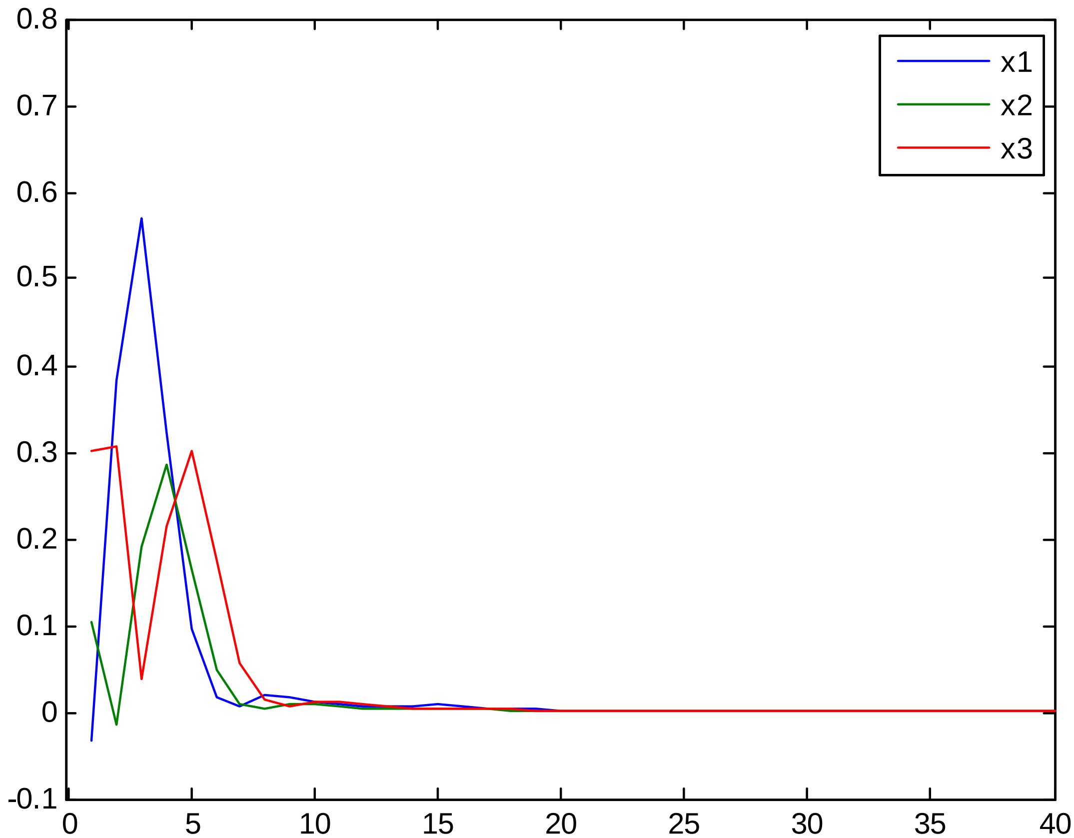

FIGURE 4.11. Observer-based H_∞ controller design by approach (4.47) with $\bar{\alpha} = 0.5$ and $\bar{\beta} = 0.4$

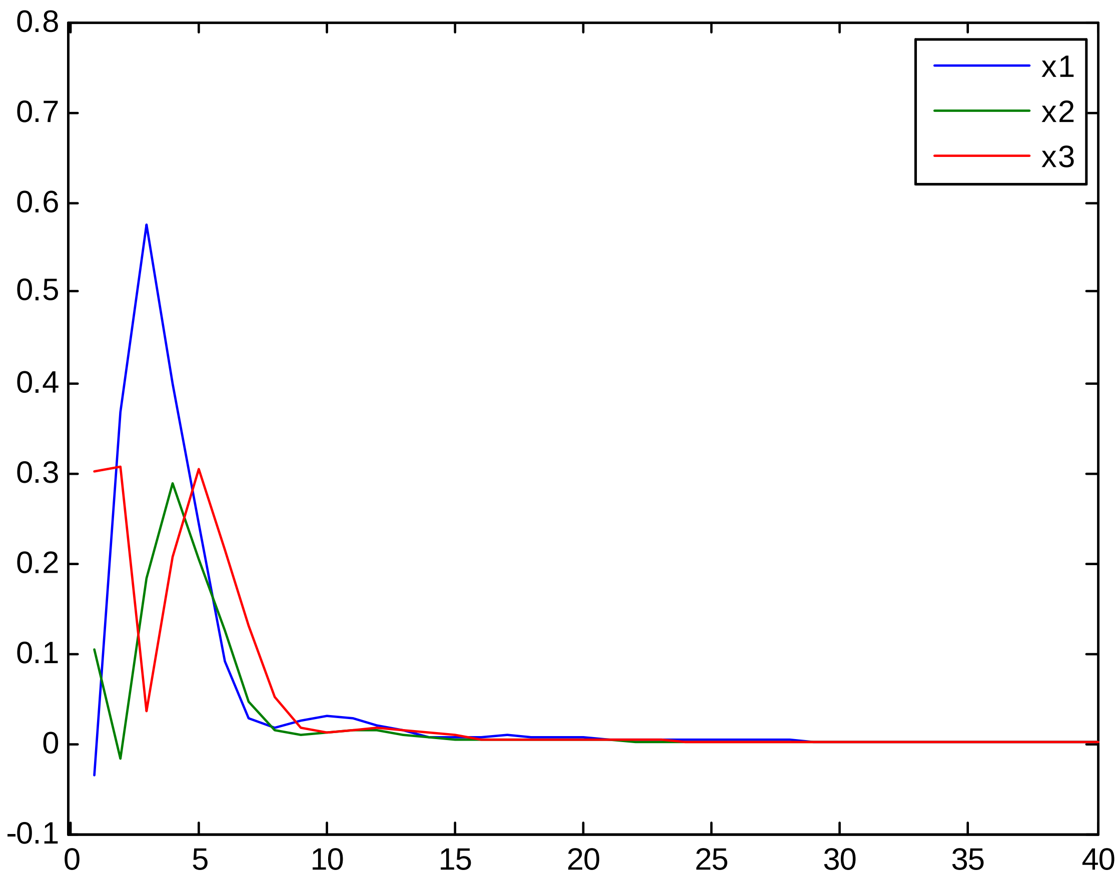

FIGURE 4.12. Observer-based H_∞ controller design by approach (4.47) with $\bar{\alpha} = 0.02$ and $\bar{\beta} = 0.1$

Chapter 5

CONLCUSIONS AND THE FUTURE WORKS

The researches in the field for the NCS are spread in vast areas. It is filled with issues and branches. The concentration here was in one of the major problems of the NCS that is the packet dropouts in the communication between the controlled plant and the controller. There were different methods for solving this problem. The work in this thesis has been considered as improvement steps for the ways of finding the linear matrix inequality approaches for the observer based controller. The observer based method is one of the famous methods in finding the solutions for NCS problem. However the researches that have been made based on this method, even though they were valid but need more simplification, and there was a demand to improve the quality of these solutions.

In this thesis, successful approaches for linear matrix inequality have been found and tested. Even though they have succeeded, more solutions should be found for the future work.

5.1 Summary with Observations

A summary of the work in this thesis, with the observations is shown in the following paragraphs.

In general, the models here were considered as nonlinear. They had white stochastic variables simulation for packet dropouts as a single packet dropout in each channel . The idea here was that the majority packet losses inside the NCS and solving it would give guarantee for the systems stability. It has been considered that the quantization problems could be found by other measurement media and tools. The systems here have been studied after the sampling. It has been considered that the packet losses for different directions of the communication would be represented by independent stochastic variables .

In chapter 3, the work was a reproduction with improvement for the work of Jain. and the others in [4] . It has been noticed that in their model there was a need for finding equality in order to use their LMI model. They also suggested that the used positive definite matrixes of their LMI approach have been considered the same but in reality they were different. The modifications for their model in this chapter has been successfully made. The new created LMI models, which have been mentioned in **??**, have succeeded because they have had the following advantages : less conservative, easier in the derivation, higher quality in the stability and perform a H_∞ minimization factor better than LMI approaches in [4]. For the future work the quantization may be added to the problem. It is also valid to add the communication delay factor in the model.

In chapter 4, the work was a mixing between three sources [4], [5] and [51]. The uncertainties have been added here for more investigation in the previous LMI approaches. The successful results here was that the LMI approaches in 4.3.1 and in 4.3.2 can make a flexibility between either making a very low minimization factor of H or minimizing the reservations in the models. They have also become easier in the use for a logic solver program with a linear type. More balanced energy consumptions have been made. The used numerical example was the same one used in the previous chapter to ensure the improvements of its LMI even though there were uncertainties added in its matrices.

The work, that has been mentioned in Appendix A, was a try to create LMI approach for a special type of NNCS with uncertainty. It is $Af(k, x_K)$ type model. Its was based on the idea of source [51], which did not include the proof of its method. Some conditions are added in this model with some suggestions that leads to LMI approaches mentioned in A.3.1 and A.3.2 . It still needs more investigation for the conditions and the suggestions validity. It requires a suitable numerical example for its conditoins.

5.2 Future Work

There are few suggestions to be added in this work for the future, which can be summarized in the following points.

1- In the controller and observer equations, the uncertainty may be included inside them for the future works. They can simulate the uncertainty in the feed back gains of the systems.

2- Finding for more and easier conditions to approximate the nonlinear state variable of the work in appendix A.

3- A suitable numerical example for the suggested conditions for the LMI in appendix A may be included

4-It is suggested to find a real application to test the LMI approaches that are created in the previous chapters.

At the end the LMI approach for NCS is still under improvements and the different type of models for it are suggested. The future is a roud for more technology that may add more flexibility and strength for the used system .

NOMENCLATURE

Abbreviations

BMI is an abbreviation for Bilinear Matrix Inequality.

DMVT is an abbreviation for Differential Mean Value Theorem.

FD is an abbreviation for Fault Detection.

FEC is an abbreviation for Forward Error Correction

LMI is an abbreviation for Linear Matrix Inequality.

LLM is an abbreviation for Laypnuov Like Method.

LQG is an abbreviation for Linear Quadratic Gaussian.

MPC is an abbreviation for Model Predictive Control .

NCS is an abbreviation for Network Control Systems.

NDC is an abbreviation for Network Delay Compensation.

NNCS is an abbreviation for Nonlinear Netwrok Control System

QoS is an abbreviation for the Quality of the Services.

QoC is an abbreviation for the Quality of the Control.

TCP is an abbreviation for the Transmission of Control Protocol

Meanings of the Symbols

- A state matrix.
- B input matrix of the plant.
- C_1output matrix of the controlled plant.
- C_2 output matrix of the measurement equation.
- D disturbance matrix of the state equation.
- D_1 disturbance matrix of the out equation.
- $f(k, x_k)$ discrete nonlinear vector variable function .
- u_k discrete control input vector,
- V_k discrete out put of the Lyapunov function
- w_k discrete disturbance input.
- x_k discrete state vector,
- y_k discrete out vector of the measurement equation.
- z_k discrete output vector of the plant.

- $\Delta A, \Delta B ...etc$ delta matrices indicate for uncertainty for matrices $A, B ...etc$
- ΔV_k derivative of the Lyapunov function
- $\Pr\{.\}$ probability of the event ".".
- E$\{x\}$ expectation of x.
- $l_2[0, \infty)$ space of square integral vectors.
- $\mathbb{I}^+$ set of the positive integers.
- $\mathbb{R}$ real numbers.
- $\mathbb{R}^n$ n-dimensional of Euclidean space.
- $\mathbb{R}^{n\times m}$ set of all $n \times m$ real materices.
- λ eigen valueof a matrix.
- $\|\cdot\|$ norm of the vector $\cdot$ or the induced matrix $2 - norm.$
- I identity matrix .
- $diag(a_1, a_2, ...a_n)$ block-diagonal matrix. with elements a_k in its main daigonal $for\ all\ \ k \in \mathbb{I}^+$.

Appendix

THE OBSERVER-BASED H_∞ CONTROLLER FOR UNCERTAIN NONLINEAR NCS WITH RANDOM PACKET LOSSES SECOND CLASS (UNDER WORK)

A.1 Introduction

This part is a trail to make observer-based H_∞ controller with uncertainty for another NNCS models. This model is different than the previous models in the chapter (4). The uncertainty in this model has been considered in the state matrix which can be separated as constant matrix invariant time to the nonlinear state variables vector and there are no partial linear variables in the system. The model of LMI has considered the packet loss for this system. It was based on the sources [51], [50] and [5] by mixing the ideas between them. The uncertainty has been applied in the states matrix of the systems and the input matrix without considering any uncertainty in the measuring equation. In sources [5] and [50] the model was applied on linear NCS but our consideration has been applied in NNCS. The NNCs that was considered here is totally nonlinear in its state matrix but in a special form. It is supposed that the system will be after the sampling. The discrete system that is considered here has one transmitting data packet and the same length for its sent data packets. The delay time has not been included here. In source [51] multiple communication packet loss has been considered but in our chapter it has been considered as one packet losses based on [44]. Multiple communication packet losses have made in source [51] for the connection between controllers to actuator different than that for the sensor to the controller and our approach will be considered as the same thing. The same method will be followed in finding the LMI approach by putting certain condition that may help in deriving the suitable theorems for this model.

A.2 Problem Formulation

The controlled plant here is a nonlinear system and has a state matrix can be taken as linear invariant time factor multiplied to the nonlinear state variables vector. The random packet losses have been considered to occur, simultaneously, in the communication channels both from the sensor to the controller and from the controller to the actuator. Similar to [4], it has been supposed that the data transmitted in single-packet manner with the same transmission length. It has been also considered that the communication type is a point-to-point network allowable data dropout rate and the point-to-point network throughout to evaluate the QoS of the investigated networked nonlinear system.

The same thing that was done in the previous chapters, it has supposed that obtaining the sampling of the system can be found by other way that is not considered here. The following networked nonlinear control system has been considered after sampling:

$$\begin{aligned} x_{k+1} &= (A + \Delta A)\, f(x_k) + (B + \Delta B)u_k + Dw_k \\ z_{k+1} &= (C_1 + \Delta C_1)x_k + D_1 w_k \end{aligned} \tag{A.1}$$

Where

$u_k \in \mathbb{R}^m$ is the control input vector,$z_k \in \mathbb{R}^r$ is the controlled output vector,$w_k \in \mathbb{R}^q$ is the disturbance input belong to $l_2[0, \infty)$, $A\&\Delta A \in \mathbb{R}^{n\times n}, B\&\Delta B \in \mathbb{R}^{n\times m}, D \in \mathbb{R}^{n\times q}, C_1\&\Delta C_1 \in \mathbb{R}^{r\times n}, D_1 \in \mathbb{R}^{r\times q}$ are known constant matrices and $f(x_k) \in \mathbb{R}^n$ is nonlinear state varaible vector.

Remark A.2.1. *It has been suggested for $f(x_k)$ for state equation satisfies the comparison lemma for the local Lipschitz conditions[52]:*

$$f(x)^T f(x) \leqslant \varphi\,(x)^T\, G^T G \varphi\,(x) \tag{A.2}$$

$$f(\hat{x})^T f(\hat{x}) \leqslant \varphi(\hat{x})^T G^T G \varphi(\hat{x}) \tag{A.3}$$

where G is known real constant matrix , and $\varphi(x)$ and $\varphi(\hat{x}) \in J \subset \mathbb{R}^+$ is upper positive nonlinear state variable with appropriate dimensions in the interval $[t_i, T_o)$ and T_o can be ∞. It has also been suggested to make $f(x)$ Global Lipschitz conditions which have been used in the previous chpaters. These conditions have been shown as

$$\|f(k,x)\| \leqslant \|Gx\| \tag{A.4}$$

$$\|f(k,x) - f(k,y)\| \leqslant \|G(x-y)\| \tag{A.5}$$

Remark A.2.2. *The suggestion has been to make the derivation of the model trail by using the second suggested conditions (A.4) and (A.5).*

The uncertainty matrices can also be described by the following limits

$$\begin{gathered} [\Delta A \;\; \Delta B\,] = M_c\, \Delta_k\, [N_1\; N_2] \\ \Delta C_1 = N_3 \Delta_k^T M_c^T \end{gathered} \tag{A.6}$$

where M_c , N_1, N_2 and N_3 are real known constant matrices with appropriate dimensions. Δ_k is unknown real matrix varying with time and has the constraint

$$\Delta_k \Delta_k^T \ < I \tag{A.7}$$

Remark A.2.3. *The considered networked control system (A.1) is a nonlinear system. The stability analysis and controller synthesis for networked nonlinear system (A.1) with random packet losses has been applied with the uncertainty. The uncertainty has been considered in the plant matrices.The feedback equation has not been included in the uncertainty and it has been supposed that the uncertainty of it is merged with its stochastic varaible .*

Remark A.2.4. *The work here has been built based on the idea that Δ_k is a square matrix.*

The measurment with random communication packet loss has been described by

$$\hat{y}_k = \alpha_k C_2 f(x_k) + D_2 w_k \tag{A.8}$$

where $\widehat{y}_k \in \mathbb{R}^p$ is the measured output vector,$C_2 \in \mathbb{R}^{p\times n}$, $D_2 \in \mathbb{R}^{p\times q}$ are real constant real matrices.The stochastic varaible $\alpha_k \in \mathbb{R}$ is a Bernoulli distributed white sequence with the properties

$$\begin{aligned}
\Pr\{\alpha_k &= 1\} = \mathbb{E}x\{\alpha_k\} = \bar{\alpha} && \text{(A.9)}\\
\Pr\{\alpha_k &= 0\} = 1 - \mathbb{E}x\{\alpha_k\} = 1 - \bar{\alpha} && \text{(A.10)}\\
var\{\alpha_k\} &= \mathbb{E}x\{(\alpha_k - \bar{\alpha})^2\} = (1-\bar{\alpha})\bar{\alpha} = \alpha_1^2 && \text{(A.11)}
\end{aligned}$$

Remark A.2.5. *α_k is linear stochastic variable that has been considered to simulate the packet dropout from the sensor to the controller.*

The dynamic observer-based control scheme for the networked nonlinear system (A.1) is described by Observer:

$$\begin{gathered}
\hat{x}_{k+1} = (A + \Delta A)\,\hat{f(x_k)} + (B + \Delta B)\bar{u}_k + L(\hat{y}_k - \bar{\alpha}C_2\hat{f(x_k)})\\
\bar{u}_k = \bar{\beta}\hat{u}_k
\end{gathered} \tag{A.12}$$

Controller

$$\begin{gathered}
\hat{u}_k = -K\hat{f(x_k)}\\
u_k = \beta_k\hat{u}_k
\end{gathered} \tag{A.13}$$

Where $\hat{f(x_k)} \in \mathbb{R}^n$ is the nonlinear state estimate of the networked non linear system (A.1),$\bar{u}_k \in \mathbb{R}^m$ is the control input of the observer,$\hat{u}_k \in \mathbb{R}^m$ is the control input without random packet loss,$u_k \in \mathbb{R}^m$ is the control input of the controlled system, $L \in \mathbb{R}^{n\times p}$ is the observer gain and $K \in \mathbb{R}^{m\times n}$is the controller gain. L and K are the two parameters to be determined. The stochastic variable $\beta_k \in \mathbb{R}$ is a Bernoulli distributed white sequence with

$$\mathrm{Pr}\{\beta_k = 1\} = \mathbb{E}x\{\alpha_k\} = \bar{\beta} \tag{A.14}$$

$$\mathrm{Pr}\{\beta_k = 0\} = 1 - \mathbb{E}x\{\alpha_k\} = 1 - \bar{\beta} \tag{A.15}$$

$$var\{\beta_k\} = \mathbb{E}x\{(\beta_k - \bar{\beta})^2\} = (1-\bar{\beta})\bar{\beta} = \beta_1^2 \tag{A.16}$$

Remark A.2.6. *β_k is linear stochastic variable which has been considered to simulate the packet dropout from the controller to the actuator.*

The random packet-loss model in (A.8) and (A.13) are built on the same idea that is shown in figure (3.1),the control input of the observer $\bar{u}_k$ is differenet from the control input of the controlled system u_k because of the existence of the random packet losses in the communication channel from the controller to the actuator. Now the state estimation error will be defined in the following:

$$e_k = f\left(x_k\right) - f\left(\hat{x}_k\right) \tag{A.17}$$

substitute (A.8) and (A.12) with (A.13) into (A.1) and (A.17) as shown in the following

First substitute $\hat{u}_k = -K(f(\hat{x}_k))$ in $\bar{u}_k = \bar{\beta}\hat{u}_k$ and in $u_k = \beta_k \hat{u}_k$

$$\begin{aligned} \bar{u}_k &= -\bar{\beta} K(f(\hat{x}_k)) \\ u_k &= -\beta_k K(f(\hat{x}_k)) \end{aligned}$$

The state equation has been represented by the following form.

$$x_{k+1} = (A + \Delta A)\, f(x_k) + (B + \Delta B) u_k + D w_k$$

It can be rewritten after subtituting $u_k = -\beta_k K(f(\hat{x}_k))$ in it as the following

$$x_{k+1} = (A + \Delta A)\, f(x_k) - \beta_k (B + \Delta B) K \; f(\hat{x}_k) + D w_k$$

From (A.17)$f(\hat{x}_k) = f(x_k) - e_k$ So

$$\begin{aligned} x_{k+1} &= (A + \Delta A)\, (f(x_k) - e_k) - \beta_k (B + \Delta B) K \; (f(x_k) - e_k) + D w_k \\ &= ((A + \Delta A) - \beta_k (B + \Delta B) K) f(x_k) + (\beta_k (B + \Delta B) K)\, e_k + D w_k \end{aligned}$$

$$
\begin{aligned}
&= ((A+\Delta A)-\beta_k(B+\Delta B)K)f(x_k)+\beta_k(B+\Delta B)Ke_k+Dw_k \\
&\quad +\bar{\beta}\left(B+\Delta B\right)Kf(x_k)-\bar{\beta}\left(B+\Delta B\right)Kf(x_k)+\bar{\beta}\left(B+\Delta B\right)Ke_k \\
&\quad -\bar{\beta}\left(B+\Delta B\right)Ke_k
\end{aligned}
$$

$$
\begin{aligned}
&= ((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)+(\beta_k-\bar{\beta})\left(B+\Delta B\right)Ke_k \\
&\quad -(\beta_k-\bar{\beta})\left(B+\Delta B\right)Kf(x_k)+\bar{\beta}\left(B+\Delta B\right)Ke_k+Dw_k
\end{aligned}
$$

Aslo $\varepsilon_{k+1}=x_{k+1}-\hat{x}_{k+1}$ and

$$
\begin{aligned}
\hat{x}_{k+1} &= (A+\Delta A)\,f(\hat{x}_k)+(B+\Delta B)\bar{u}_k+L(\hat{y}_k-\bar{\alpha}C_2f(\hat{x}_k)) \\
&= (A+\Delta A)f(\hat{x}_k)-\bar{\beta}(B+\Delta B)Kf(\hat{x}_k)+L(\hat{y}_k-\bar{\alpha}C_2f(\hat{x}_k)) \\
&= \big((A+\Delta A)-\bar{\beta}(B+\Delta B)K\big)\,f(\hat{x}_k)+L(\hat{y}_k-\bar{\alpha}C_2f(\hat{x}_k))
\end{aligned}
$$

It can be rewritten in the following form

$$
\begin{aligned}
\varepsilon_{k+1} &= [((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)+(\beta_k-\bar{\beta})\left(B+\Delta B\right)Ke_k \\
&\quad -(\beta_k-\bar{\beta})\left(B+\Delta B\right)Kf(x_k)+\bar{\beta}\left(B+\Delta B\right)Ke_k+Dw_k] \\
&\quad -[\big((A+\Delta A)-\bar{\beta}(B+\Delta B)K\big)\,f(\hat{x}_k)+L(\hat{y}_k-\bar{\alpha}C_2f(\hat{x}_k))]
\end{aligned}
$$

$$
\begin{aligned}
= \;& [((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k \\
& -(\beta_k-\bar{\beta})(B+\Delta B)Kf(x_k) \\
& +\bar{\beta}(B+\Delta B)Ke_k+Dw_k] \\
& -[((A+\Delta A)-\bar{\beta}(B+\Delta B)K)(f(x_k)-e_k) \\
& +L(\alpha_k C_2 f(x_k)+D_2 w_k-\bar{\alpha}C_2(f(x_k)-e_k)]
\end{aligned}
$$

$$
\begin{aligned}
= \;& [((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k \\
& -(\beta_k-\bar{\beta})(B+\Delta B)Kf(x_k) \\
& +\bar{\beta}(B+\Delta B)Ke_k+Dw_k] \\
& -[(((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k) \\
& -((A+\Delta A)-\bar{\beta}(B+\Delta B)K)e_k) \\
& +(\alpha_k LC_2 f(x_k)+LD_2 w_k-(\bar{\alpha}LC_2 f(x_k)-\bar{\alpha}LC_2 e_k)]
\end{aligned}
$$

$$
\begin{aligned}
&= ((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k \\
&\quad -(\beta_k-\bar{\beta})(B+\Delta B)Kf(x_k) \\
&\quad +\bar{\beta}(B+\Delta B)Ke_k+Dw_k-((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k) \\
&\quad +((A+\Delta A)-\bar{\beta}(B+\Delta B)K)e_k)-\alpha_k LC_2 f(x_k) \\
&\quad -LD_2 w_k+\bar{\alpha}LC_2 f(x_k)-\bar{\alpha}LC_2 e_k \\
&= -(\beta_k-\bar{\beta})(B+\Delta B)Kf(x_k)-(\alpha_k-\bar{\alpha})LC_2 f(x_k)+((A+\Delta A)-\bar{\alpha}LC_2)e_k \\
&\quad +(\beta_k-\bar{\beta})(B+\Delta B)Ke_k+(D-LD_2)w_k
\end{aligned}
$$

This has resulted in the following closed-loop netwroked nonlinear system:

$$
\begin{gathered}
x_{k+1}=((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k \\
-(\beta_k-\bar{\beta})(B+\Delta B)Kf(x_k)+\bar{\beta}(B+\Delta B)Ke_k+Dw_k \\
\varepsilon_{k+1}=-(\beta_k-\bar{\beta})(B+\Delta B)Kf(x_k)-(\alpha_k-\bar{\alpha})LC_2 f(x_k)+((A+\Delta A)-\bar{\alpha}LC_2)e_k \\
+(\beta_k-\bar{\beta})(B+\Delta B)Ke_k+(D-LD_2)w_k
\end{gathered}
\tag{A.18}
$$

Then, it can be rewritten in other form by using the following notation.

$$
\eta_k=\begin{bmatrix} f(x_k) \\ e_k \end{bmatrix} \tag{A.19}
$$

As a result the closed nonlinear system in (A.18) can be described in the following compact form

$$\eta_{k+1} = \bar{A}\eta_k + (\beta_k - \bar{\beta})\hat{A}_1\eta_k + (\alpha_k - \bar{\alpha})\hat{A}_2\eta_k + \bar{B}w_k \quad \text{(A.20)}$$

where $\bar{A} = \begin{bmatrix} (A+\Delta A) + \bar{\beta}(B+\Delta B)K & \bar{\beta}(B+\Delta B)K \\ 0 & (A+\Delta A) - \bar{\alpha}LC_2 \end{bmatrix}$,

$$\hat{A}_1 = \begin{bmatrix} -(B+\Delta B)K & (B+\Delta B)K \\ -(B+\Delta B)K & (B+\Delta B)K \end{bmatrix}, \hat{A}_2 = \begin{bmatrix} 0 & 0 \\ -LC_2 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} D \\ D - LD_2 \end{bmatrix}.$$

This closed nonlinear system netwrok ,contains the sotchastic parameteres α_k, β_k .

Then it is needed to ensure its stability.

The main purposes can be summarized in two points.

1-The closed -loop nonlinear networked system (A.18) is stable.

2-Under the zero-intial condition , for all nonzero w_k the controlled output z_k should satisfy

$$\sum_{k=0}^{\infty} \mathbb{E}\left\|z_k\right\|^2 < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left\|w_k\right\|^2 \quad \text{(A.21)}$$

where $\gamma > 0$ is a prescribed scalar scalar.

A.3 Main Results

The same lemmas that were used in chapter 4 have been used here. Without loss of generality, we first introduced the following lemmas for technical convenience.

Lemma A.3.1. *([4,41])(S-procedure)). Let* $T_i \in \mathbb{R}^{n\times n}(i = 0, 1, 2, ..., p)$ *be symmetric matrices. The conditions on* T_i $(i = 0, 1, 2, ..., p)$ $\varsigma^T T_o \varsigma > 0, \quad \forall \varsigma \neq 0 \; s.t. \varsigma^T T_i \varsigma \geq 0 \;\; (i = 0, 1, 2, ..., p)$ *hold if there exists* $\tau_i \geq 0$ $(i = 0, 1, 2, ..., p)$ *such that* $T_o - \sum_{i=1}^{p} \tau_i T_i > 0$.

Lemma A.3.2. *([5,41]) Let* $M = M^T$ *and H and E be real matrices of appropraite dimensions with F satisfying* $FF^T < I \;\; then \;\; M + HFE + E^T F^T H^T < 0$, *if and only if there exisits a postive scalar* $\epsilon > 0$ *such that*

$$M + \frac{1}{\epsilon} HH^T + \epsilon E^T E < 0 \tag{A.22}$$

or equivalently

$$\begin{bmatrix} M & H & \epsilon E^T \\ H^T & -\epsilon I & 0 \\ \epsilon E & 0 & -\epsilon I \end{bmatrix} < 0 \tag{A.23}$$

In the following theorem, we will derive a sufficient condition such that the closed-loop networked nonlinear system (A.20) is stable.

Theorem A.3.1. *Gven the communication channel parameters* $0 \leq \bar{\alpha} \leq 1,\ 0 \leq \bar{\beta} \leq 1$ *, the controller gain matrix* K *and the observer gain matrix* L*. The closed-loop networked nonlinear system (A.20) is exponentially mean square stable if there exists positive-definite matrices* $P,\ Q$*, and nonnegative real scalars* $\tau_1 > 0,\ \tau_2 > 0$ *with the uncertainty that is mentioned in (A.6) and (A.7) satisfying the linear matrix inequality as shown in the following*

$$\begin{bmatrix} \hat{\Lambda}_8 & \tilde{\Lambda}_8 & \breve{\Lambda}_8 \end{bmatrix} < 0 \tag{A.24a}$$

where

$$\hat{\Lambda}_8 = \begin{bmatrix} -P & * & * & * \\ 0 & -\tau_1 I & * & * \\ 0 & 0 & -Q & * \\ \tau_1^{0.5} G & 0 & 0 & -I \\ 0 & P(A-\bar{\beta}BK) & \bar{\beta}PBK & 0 \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 \\ 0 & \beta_1 QBK & -\beta_1 QBK & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 \\ 0 & 0 & Q(A-\bar{\alpha}LC_2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_8 = \begin{bmatrix} -P & * & * & * \\ 0 & -\tau_1 I & * & * \\ 0 & 0 & -Q & * \\ \tau_1^{0.5} G & 0 & 0 & -I \\ 0 & P(A-\bar{\beta}BK) & \bar{\beta}PBK & 0 \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 \\ 0 & \beta_1 QBK & -\beta_1 QBK & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 \\ 0 & 0 & Q(A-\bar{\alpha}LC_2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\breve{\Lambda}_8 = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ -P & * & * & * \\ 0 & -P & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -Q \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (PM_c)^T & \beta_1 (PM_c)^T & \beta_1 (QM_c)^T & 0 \\ \bar{\beta} (PM_c)^T & -\beta_1 (PM_c)^T & -\beta_1 (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 \\ \left(N_1 - \bar{\beta} N_2 K\right) & (N_2 K) & (N_2 K) & 0 \\ (N_2 K) & N_2 K & (N_2 K) & 0 \end{bmatrix}$$

, $\alpha_1 = [(1-\bar{\alpha})\bar{\alpha}]^{1/2}\, and\ \ \beta_1 = \left[(1-\bar{\beta})\bar{\beta}\right]^{1/2}$.

Proof. By using a Lyaqunov function

$$V_k = x_k^T P x_k + e_k^T Q e_k \tag{A.25}$$

where $P,\ Q$ *are positive definite matrices solution to*

$$\Delta V_k = \mathbb{E}\{V_{k+1} | x_k, x_{k-1}, x_{k-2}, ..., x_0, e_k, e_{k-1}, ..., e_0\} - V_k$$

$$\Delta V_k = \mathbb{E}\{x_{k+1}^T P x_{k+1} + \varepsilon_{k+1}^T Q \varepsilon_{k+1}\} - x_k^T P x_k - e_k^T Q e_k$$

$$\Delta V_k = \mathbb{E}\left\{[V_1]^T P [V_1] + [V_2]^T Q [V_2]\right\} - x_k^T P x_k - e_k^T Q e_k^T \tag{A.26}$$

where

$$V_1 = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k) + (\beta_k - \bar{\beta})(B + \Delta B)Ke_k$$

$$-(\beta_k - \bar{\beta})(B + \Delta B)Kf(x_k) + \bar{\beta}(B + \Delta B)Ke_k$$

$$V_2 = -(\beta_k - \bar{\beta})(B + \Delta B)Kf(x_k) - (\alpha_k - \bar{\alpha})LC_2 f(x_k)$$

$$+((A + \Delta A) - \bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})(B + \Delta B)Ke_k$$

For the simpilicity in deriving the equations the following to be assumed

$$T_1 = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k) + \bar{\beta}(B + \Delta B)Ke_k$$

$$T_2 = (B + \Delta B)Ke_k - (B + \Delta B)Kf(x_k)$$

$$T_3 = ((A + \Delta A) - \bar{\alpha}LC_2)e_k$$

$$T_4 = LC_2 f(x_k)$$

So ΔV_k can be rewritten as following

$$\Delta V_k = \mathbb{E}\left\{T_1^T P T_1 + (\beta_k - \bar{\beta})T_2^T P T_1\right\} + \mathbb{E}\left\{(\beta_k - \bar{\beta})T_1^T P T_2 + (\beta_k - \bar{\beta})^2 T_2^T P T_2\right\}$$

$$+\mathbb{E}\left\{T_3^T Q T_3 - (\alpha_k - \bar{\alpha})T_4^T Q T_3 + (\beta_k - \bar{\beta})T_2^T Q T_3\right\}$$

$$-\mathbb{E}\left\{(\alpha_k - \bar{\alpha})T_3^T Q T_4 - (\alpha_k - \bar{\alpha})^2 T_4^T Q T_4 + (\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta})T_2^T Q T_4\right\}$$

$$+\mathbb{E}\left\{(\beta_k-\bar{\beta})T_3^TQT_2-(\beta_k-\bar{\beta})(\alpha_k-\bar{\alpha})T_4^TQT_2+(\beta_k-\bar{\beta})^2T_2^TQT_2\right\}$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

Based on the property of Bernoulli stochastic varaible and the independence, $\mathbb{E}\{(\beta_k-\bar{\beta})\}=\mathbb{E}(\beta_k)-\bar{\beta}=\bar{\beta}-\bar{\beta}=0$, *and the same thing for* $\mathbb{E}\{(\alpha_k-\bar{\alpha})\}=0$ *Aslo,*$\mathbb{E}\{(\beta_k-\bar{\beta})^2\}=\beta_1^2$ *and* $\mathbb{E}\{(\alpha_k-\bar{\alpha})^2\}=\alpha_1^2$*. Then* $\Delta V_k=\left[T_1^TPT_1\right]+\left[\mathbb{E}\{(\beta_k-\bar{\beta})^2\}T_2^TPT_2\right]+\left[T_3^TQT_3\right]$

$$-\left[-\mathbb{E}\{(\alpha_k-\bar{\alpha})^2\}T_4^TQT_4+\mathbb{E}\{(\alpha_k-\bar{\alpha})(\beta_k-\bar{\beta})\}T_2^TQT_4\right]$$

$$+\left[-\mathbb{E}\{(\beta_k-\bar{\beta})(\alpha_k-\bar{\alpha})\}T_4^TQT_2+\mathbb{E}\{(\beta_k-\bar{\beta})^2\}T_2^TQT_2\right]$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

From (A.11) and (A.16) ΔV_k *can be written in the following form.*

$$\Delta V_k=\left[T_1^TPT_1\right]+\left[\beta_1^2T_2^TPT_2\right]+\left[T_3^TQT_3\right]$$

$$-\left[-\alpha_1^2T_4^TQT_4+\mathbb{E}\{(\alpha_k-\bar{\alpha})(\beta_k-\bar{\beta})\}T_2^TQT_4\right]$$

$$+\left[-\mathbb{E}\{(\beta_k-\bar{\beta})(\alpha_k-\bar{\alpha})\}T_4^TQT_2+\beta_1^2T_2^TQT_2\right]$$

$$-x_k^TPx_k-e_k^TQe_k^T$$

Also, $\mathbb{E}\{(\alpha_k-\bar{\alpha})(\beta_k-\bar{\beta})\}=\mathbb{E}\{(\alpha_k\beta_k-\bar{\alpha}\beta_k-\alpha_k\bar{\beta}+\bar{\alpha}\bar{\beta})\}=\mathbb{E}\{(\alpha_k\beta_k)\}-\mathbb{E}\{\bar{\alpha}\beta_k\}-\mathbb{E}\{\alpha_k\bar{\beta}\}+\mathbb{E}\{(\bar{\alpha}\bar{\beta})\}$*.This is done because the property of the Bernoulli propabillity and the independence property of them. This has resulted in this equation* $\mathbb{E}\{(\alpha_k\beta_k)\}-\mathbb{E}\{\bar{\alpha}\beta_k\}-$

$\mathbb{E}\{\alpha_k\bar{\beta}\} + \mathbb{E}\{(\bar{\alpha}\bar{\beta})\} = \bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta} + \bar{\alpha}\bar{\beta} = 0.$ $\Delta V_k = \left[T_1^T P T_1\right] + \left[\beta_1^2 T_2^T P T_2\right] + \left[T_3^T Q T_3\right] - \left[-\alpha_1^2 T_4^T Q T_4\right] + \left[\beta_1^2 T_2^T Q T_2\right]$

$$-x_k^T P x_k - e_k^T Q e_k^T$$

$$\Delta V_k = \left[T_1^T P T_1\right] + \left[\beta_1^2 T_2^T P T_2\right] + \left[T_3^T Q T_3\right] + \alpha_1^2 T_4^T Q T_4 + \left[\beta_1^2 T_2^T Q T_2\right]$$

$$-x_k^T P x_k - e_k^T Q e_k^T$$

Then, the following equation has been substituted.

$$T_1 = ((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K) f(x_k) + \bar{\beta}\,(B + \Delta B)\,K e_k$$

$$T_2 = (B + \Delta B)\,K e_k - (B + \Delta B)\,K f(x_k)$$

$$T_3 = ((A + \Delta A) - \bar{\alpha} L C_2) e_k$$

$$T_4 = L C_2 f(x_k)$$

This has resulted in the following equation.

$$\Delta V_k = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 - {x_k}^T P x_k - e_k^T Q e_k^T$$

where

$$V_1 = (((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K) f(x_k))^T P((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K) f(x_k)$$

$$+\bar{\beta}((B + \Delta B)\,K e_k)^T P((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K) f(x_k))$$

$$V_2 = (((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K) f(x_k))^T \bar{\beta} P\,(B + \Delta B)\,K e_k$$

$$+\bar{\beta}((B + \Delta B)\,K e_k)^T \bar{\beta} P\,(B + \Delta B)\,K e_k$$

$$V_3 = \beta_1^2\,(B+\Delta B)\,Ke_k)^T P\,(B+\Delta B)\,Ke_k$$

$$-\beta_1^2\,(((B+\Delta B)\,Kf(x_k))^T\,P\,(B+\Delta B)\,Ke_k$$

$$-\beta_1^2\,(B+\Delta B)\,Ke_k)^T P\,(B+\Delta B)\,Kf(x_k)$$

$$+\beta_1^2\,(((B+\Delta B)\,Kf(x_k))^T\,P\,(B+\Delta B)\,Kf(x_k)$$

$$V_4 = (((A+\Delta A)-\bar{\alpha}LC_2)e_k)^T Q((A+\Delta A)-\bar{\alpha}LC_2)e_k$$

$$V_5 = \alpha_1^2\,(LC_2f(x_k))^T\,(QLC_2f(x_k))$$

$$V_6 = \beta_1^2\,(B+\Delta B)\,Ke_k)^T Q\,(B+\Delta B)\,Ke_k$$

$$-\beta_1^2\,(((B+\Delta B)\,Kf(x_k))^T\,Q\,(B+\Delta B)\,Ke_k$$

$$-\beta_1^2\,(B+\Delta B)\,Ke_k)^T Q\,(B+\Delta B)\,Kf(x_k)$$

$$+\beta_1^2\,(((B+\Delta B)\,Kf(x_k))^T\,Q\,(B+\Delta B)\,Kf(x_k)$$

This has resulted in the following matrices multiplication

$$\Delta V_k = \begin{bmatrix} x_k \\ f(x_k) \\ e_k \end{bmatrix}^T \Lambda \begin{bmatrix} x_k \\ f(x_k) \\ e_k \end{bmatrix} \triangleq \xi_k^T \Lambda \xi_k \qquad \text{(A.27)}$$

where

$$\Lambda = \begin{bmatrix} -P & 0 & 0 \\ 0 & \Phi_{1\,1} & * \\ 0 & \Phi_{2\,1} & \Phi_{2\,2} \end{bmatrix}$$

$$\Phi_{1\,1} = ((A+\Delta A)-\bar{\beta}\,(B+\Delta B)\,K)^T P((A+\Delta A)-\bar{\beta}\,(B+\Delta B)\,K)$$

$$+\beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K + \beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$+\alpha_1^2 C_2^T L^T Q L C_2$$

$$\Phi_{2\,2} = \bar{\beta}^2 K^T (B+\Delta B)^T P (B+\Delta B) K + \beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$+\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$+((A+\Delta A) - \bar{\alpha} L C_2)^T Q ((A+\Delta A) - \bar{\alpha} L C_2) - Q$$

$$\Phi_{2\,1} = \bar{\beta} K^T (B+\Delta B)^T P ((A+\Delta A) - \bar{\beta} (B+\Delta B) K)$$

$$-\beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$-\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

It has followed from (A.4) and (A.5) that

$$f(x_k)^T f(x_k) \leqslant x_k^T G^T G x_k \tag{A.28}$$

It can be rewritten in the following inequality. $f(x_k)^T f(x_k) - x_k^T G^T G x_k \leqslant 0$

$$\xi_k^T \begin{bmatrix} -G^T G & * & * \\ 0 & I & * \\ 0 & 0 & 0 \end{bmatrix} \xi_k \triangleq \xi_k^T \Lambda_1 \xi_k \leq 0 \tag{A.29}$$

By the S-procedure $\Delta V_k = \xi_k^T \Lambda \xi_k^T < 0$ *with the constrains (A.29) holds if there exists* *positive-definite matrices* P, Q *and positive scalars* $\tau_1 > 0$, *such that*

$$\Lambda - \tau_1 \Lambda_1 < 0 \tag{A.30}$$

It can be rewritten in the following inequality

$$\Lambda - \tau_1 \Lambda_1 = \begin{bmatrix} -P + \tau_1 G^T G & 0 & 0 \\ 0 & \tilde{\Phi}_{1\,1} & * \\ 0 & \Phi_{2\,1} & \Phi_{2\,2} \end{bmatrix} < 0$$

where

$$\tilde{\Phi}_{1\,1} = ((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K)^T P((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K)$$

$$+\beta_1^2 K^T\,(B + \Delta B)^T\,P\,(B + \Delta B)\,K + \beta_1^2 K^T\,(B + \Delta B)^T\,Q\,(B + \Delta B)\,K$$

$$+\alpha_1^2 C_2^T L^T Q L C_2$$

$$-\tau_1 I$$

$$\Phi_{2\,2} = \bar{\beta}^2\,K^T\,(B + \Delta B)^T\,P\,(B + \Delta B)\,K + \beta_1^2 K^T\,(B + \Delta B)^T\,P\,(B + \Delta B)\,K$$

$$+\beta_1^2 K^T\,(B + \Delta B)^T\,Q\,(B + \Delta B)\,K$$

$$+((A + \Delta A) - \bar{\alpha} L C_2)^T Q((A + \Delta A) - \bar{\alpha} L C_2) - Q$$

$$\Phi_{2\,1} = \bar{\beta} K^T\,(B + \Delta B)^T\,P((A + \Delta A) - \bar{\beta}\,(B + \Delta B)\,K)$$

$$-\beta_1^2 K^T\,(B + \Delta B)^T\,P\,(B + \Delta B)\,K$$

$$-\beta_1^2 K^T\,(B + \Delta B)^T\,Q\,(B + \Delta B)\,K$$

or it can be in the equivalent form of a MI as shown below

$$\Lambda - \tau_1\Lambda_1 = \begin{bmatrix} -P & * & * \\ 0 & -\tau_1 I & * \\ 0 & 0 & -Q \end{bmatrix} + \begin{bmatrix} \tau_1 G^T G & 0 & 0 \\ 0 & \check{\Phi}_{1\,1} & * \\ 0 & \Phi_{2\,1} & \check{\Phi}_{2\,2} \end{bmatrix}$$

where

$$\check{\Phi}_{1\,1} = ((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K)^T P((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K)$$

$$+\beta_1^2 K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K + \beta_1^2 K^T\,(B+\Delta B)^T\,Q\,(B+\Delta B)\,K$$

$$+\alpha_1^2 C_2^T L^T Q L C_2$$

$$\check{\Phi}_{2\,2} = \bar{\beta}^2\,K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K + \beta_1^2 K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K$$

$$+\beta_1^2 K^T\,(B+\Delta B)^T\,Q\,(B+\Delta B)\,K$$

$$+((A+\Delta A) - \bar{\alpha} L C_2)^T Q((A+\Delta A) - \bar{\alpha} L C_2)$$

$$\Phi_{2\,1} = \bar{\beta} K^T\,(B+\Delta B)^T\,P((A+\Delta A) - \bar{\beta}\,(B+\Delta B)\,K)$$

$$-\beta_1^2 K^T\,(B+\Delta B)^T\,P\,(B+\Delta B)\,K$$

$$-\beta_1^2 K^T\,(B+\Delta B)^T\,Q\,(B+\Delta B)\,K$$

It can be rewritten in the following form

$$\Lambda - \tau_1\Lambda_1 = \begin{bmatrix} -P & * & * \\ 0 & -\tau_1 I & * \\ 0 & 0 & -Q \end{bmatrix} + \begin{bmatrix} U_1 & U_2 \end{bmatrix}$$

$$\times\begin{bmatrix} I^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & P^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & P^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q^{-1} \end{bmatrix}$$

$$\times\begin{bmatrix} \tau_1^{0.5}G & 0 & 0 \\ 0 & P((A+\Delta A)-\bar{\beta}\,(B+\Delta B)\,K) & \bar{\beta}P\,(B+\Delta B)\,K \\ 0 & \beta_1 P\,(B+\Delta B)\,K & -\beta_1 P\,(B+\Delta B)\,K \\ 0 & \beta_1 Q\,(B+\Delta B)\,K & -\beta_1 Q\,(B+\Delta B)\,K \\ 0 & \alpha_1 QLC_2 & 0 \\ 0 & 0 & Q((A+\Delta A)-\bar{\alpha}LC_2) \end{bmatrix}$$

where

$$U_1=\begin{bmatrix} \tau_1^{0.5}G & 0 \\ 0 & ((A+\Delta A)-\bar{\beta}\,(B+\Delta B)\,K)^T P \\ 0 & \bar{\beta}K^T\,(B+\Delta B)^T\,P \end{bmatrix}$$

$$U_2=\begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta_1 K^T\,(B+\Delta B)^T\,P & \beta_1 K^T\,(B+\Delta B)^T\,Q & \alpha_1 C_2^T L^T Q & 0 \\ -\beta_1 K^T\,(B+\Delta B)^T\,P & -\beta_1 K^T\,(B+\Delta B)^T\,Q & 0 & ((A+\Delta A)-\bar{\alpha}LC_2)^T Q \end{bmatrix}$$

It is noted that τ_1 has been put under the square root in order to make s-procedure applicable. It is scalar so it still known as a positive constant. Thus, the MI has been in the following shape

$$\Lambda-\tau_1\Lambda_1=\begin{bmatrix} \tilde{\Lambda}_1 & \Psi_1^T \\ \Psi_1 & \breve{\Lambda}_1 \end{bmatrix}$$

where

$$\tilde{\Lambda}_1 = \begin{bmatrix} -P & * & * \\ 0 & -\tau_1 I & * \\ 0 & 0 & -Q \end{bmatrix},$$

$$\breve{\Lambda}_1 = \begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -P & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} \tau_1^{0.5} G & 0 & 0 \\ 0 & P((A+\Delta A) - \bar{\beta}(B+\Delta B)K) & \bar{\beta} P(B+\Delta B)K \\ 0 & \beta_1 P(B+\Delta B)K & -\beta_1 P(B+\Delta B)K \\ 0 & \beta_1 Q(B+\Delta B)K & -\beta_1 Q(B+\Delta B)K \\ 0 & \alpha_1 QLC_2 & 0 \\ 0 & 0 & Q((A+\Delta A) - \bar{\alpha} LC_2) \end{bmatrix},$$ *It can be reconstructed in the following form*

$$\Lambda - \tau_1 \Lambda_1 = \begin{bmatrix} \tilde{\Lambda}_2 & \Psi_2^T \\ \Psi_2 & \breve{\Lambda}_2 \end{bmatrix} + \begin{bmatrix} \tilde{\Lambda}_3 & \Psi_3^T \\ \Psi_3 & \breve{\Lambda}_3 \end{bmatrix}$$

where

$$\tilde{\Lambda}_2 = \begin{bmatrix} -P & * & * \\ 0 & -\tau_1 I & * \\ 0 & 0 & -Q \end{bmatrix},$$

$$\breve{\Lambda}_2 = \begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -P & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} \tau_1^{0.5}G & 0 & 0 \\ 0 & P(A-\bar{\beta}BK) & \bar{\beta}PBK \\ 0 & \beta_1 PBK & -\beta_1 PBK \\ 0 & \beta_1 QBK & -\beta_1 QBK \\ 0 & \alpha_1 QLC_2 & 0 \\ 0 & 0 & Q(A-\bar{\alpha}LC_2) \end{bmatrix},$$

$$\tilde{\Lambda}_3 = \begin{bmatrix} 0 & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix},$$

$$\breve{\Lambda}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \textit{and}$$

$$\Psi_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (P(\Delta A) - \bar{\beta} P(\Delta B) K) & \bar{\beta} P(\Delta B) K \\ 0 & \beta_1 P(\Delta B) K & -\beta_1 P(\Delta B) K \\ 0 & \beta_1 Q(\Delta B) K & -\beta_1 Q(\Delta B) K \\ 0 & 0 & 0 \\ 0 & 0 & Q(\Delta A) \end{bmatrix}$$

Based on (A.6)

$$\Lambda - \tau_1 \Lambda_1 = \begin{bmatrix} \tilde{\Lambda}_4 & \Psi_4^T \\ \Psi_4 & \breve{\Lambda}_4 \end{bmatrix} + \begin{bmatrix} \tilde{\Lambda}_5 & \Psi_5^T \\ \Psi_5 & \breve{\Lambda}_5 \end{bmatrix}$$

where

$$\tilde{\Lambda}_4 = \begin{bmatrix} -P & * & * \\ 0 & -\tau_1 I & * \\ 0 & 0 & -Q \end{bmatrix},$$

$$\breve{\Lambda}_4 = \begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -P & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\Psi_4 = \begin{bmatrix} \tau_1^{0.5} G & 0 & 0 \\ 0 & P(A - \bar{\beta} BK) & \bar{\beta} PBK \\ 0 & \beta_1 PBK & -\beta_1 PBK \\ 0 & \beta_1 QBK & -\beta_1 QBK \\ 0 & \alpha_1 QLC_2 & 0 \\ 0 & 0 & Q(A - \bar{\alpha} LC_2) \end{bmatrix},$$

$$\tilde{\Lambda}_5 = \begin{bmatrix} 0 & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix}, \breve{\Lambda}_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & (P(M_c\Delta_k N_1) - \bar{\beta}P\left(M_c\Delta_k N_2\right)K) & \bar{\beta}P\left(M_c\Delta_k N_2\right)K \\ 0 & \beta_1 P\left(M_c\Delta_k N_2\right)K & -\beta_1 P\left(M_c\Delta_k N_2\right)K \\ 0 & \beta_1 Q\left(M_c\Delta_k N_2\right)K & -\beta_1 Q\left(M_c\Delta_k N_2\right)K \\ 0 & 0 & 0 \\ 0 & 0 & Q(M_c\Delta_k N_1) \end{bmatrix}$$

Also based on Lemma (A.22) and Lemma (A.23) it can be rewritten in the following MI.

Then $\Lambda - \tau_1\Lambda_1$ *can be rewritten in the following*

$$\Lambda - \tau_1\Lambda_1 =$$

$$\begin{bmatrix} \tilde{\Lambda}_6 & \Psi_6^T \\ \Psi_6 & \breve{\Lambda}_6 \end{bmatrix} + [\Psi_9] \begin{bmatrix} \Delta_k & 0 & 0 \\ 0 & \Delta_k & 0 \\ 0 & 0 & \Delta_k \end{bmatrix} [\Psi_{10}] + [\Psi_{10}]^T \begin{bmatrix} \Delta_k & 0 & 0 \\ 0 & \Delta_k & 0 \\ 0 & 0 & \Delta_k \end{bmatrix} [\Psi_9]^T$$

where $\tilde{\Lambda}_6 = \begin{bmatrix} -P & * & * \\ 0 & -\tau_1 I & * \\ 0 & 0 & -Q \end{bmatrix},$

$$\breve{\Lambda}_6 = \begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & -P & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\Psi_6 = \begin{bmatrix} \tau_1^{0.5}G & 0 & 0 \\ 0 & P(A-\bar{\beta}BK) & \bar{\beta}PBK \\ 0 & \beta_1 PBK & -\beta_1 PBK \\ 0 & \beta_1 QBK & -\beta_1 QBK \\ 0 & \alpha_1 QLC_2 & 0 \\ 0 & 0 & Q(A-\bar{\alpha}LC_2) \end{bmatrix},$$

$$\Psi_9 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & (PM_c) & \bar{\beta}\,(PM_c) \\ 0 & \beta_1\,(PM_c) & -\beta_1\,(PM_c) \\ 0 & \beta_1\,(QM_c) & -\beta_1\,(QM_c) \\ 0 & 0 & 0 \\ 0 & 0 & (QM_c) \end{bmatrix}, \textit{ and}$$

$$\Psi_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(N_1 - \bar{\beta}N_2K\right) & (N_2K) & (N_2K) & 0 & 0 \\ 0 & 0 & 0 & 0 & (N_2K) & N_2K & (N_2K) & 0 & N_1 \end{bmatrix}.$$

From (A.7) $(\Delta_k \Delta_k^T) < I$, *It can be held if there is* $\epsilon > 0$. *As a result of that it has* *become as the following.*

$$\Lambda - \tau_1\Lambda_1 = \begin{bmatrix} \tilde{\Lambda}_8 & \breve{\Lambda}_8 & \bar{\Lambda}_8 \end{bmatrix} < 0 \textit{ where}$$

$$\tilde{\Lambda}_8 = \begin{bmatrix} \tilde{\Lambda}_{18} \\ \tilde{\Lambda}_{28} \\ \tilde{\Lambda}_{38} \end{bmatrix},$$

$$\tilde{\Lambda}_{18} = \begin{bmatrix} -P & * & * & * \\ 0 & -\tau_1 I & * & * \\ 0 & 0 & -Q & * \\ \tau_1^{0.5}G & 0 & 0 & -I \end{bmatrix},$$

$$\tilde{\Lambda}_{28} = \begin{bmatrix} 0 & P(A-\bar{\beta}BK) & \bar{\beta}PBK & 0 \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 \\ 0 & \beta_1 QBK & -\beta_1 QBK & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_{38} = \begin{bmatrix} 0 & 0 & Q(A-\bar{\alpha}LC_2) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\breve{\Lambda}_8 = \begin{bmatrix} \breve{\Lambda}_{18} \\ \breve{\Lambda}_{28} \end{bmatrix},$$

$$\breve{\Lambda}_{18} = \begin{bmatrix} & \tilde{\Lambda}_{28}^T & & \\ -P & * & * & * \\ 0 & -P & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -Q \end{bmatrix},$$

$$\breve{\Lambda}_{28} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (PM_c)^T & \beta_1 (PM_c)^T & \beta_1 (QM_c)^T & 0 \\ \bar{\beta} (PM_c)^T & -\beta_1 (PM_c)^T & -\beta_1 (QM_c)^T & 0 \\ 0 & 0 & 0 & 0 \\ \left(N_1 - \bar{\beta} N_2 K\right) & (N_2 K) & (N_2 K) & 0 \\ (N_2 K) & N_2 K & (N_2 K) & 0 \end{bmatrix},$$

$$\bar{\Lambda}_8 = \begin{bmatrix} \bar{\Lambda}_{18} \\ \bar{\Lambda}_{28} \end{bmatrix},$$

$$\bar{\Lambda}_{18} = \tilde{\Lambda}_{38}^T$$

and

$$\bar{\Lambda}_{28} = \begin{bmatrix} -Q & * & * & * & * & * & * \\ 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & -\epsilon I & * & * & * & * \\ (QM_c)^T & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & -\epsilon I & * \\ N_1 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix}.$$

This will give the same MI shown in (A.24a). Thus, we have

$\Delta V_k = \xi_k^T \Lambda \xi_k < 0 and \ \Lambda < 0$ *that*

$\Delta V_k = \xi_k^T \Lambda \xi_k \leq -\lambda_{\min}(-\Lambda)\xi_k^T \xi_k$

$\Delta V_k \leq -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + f(x_k)^T f(x_k) + F_k^T F_k)$

$\Delta V_k \leq -\lambda_{\min}(-\Lambda)(\eta_k^T \eta_k + \|f(x_k)\|^2 + \|F_k\|^2) < -\alpha \eta_k^T \eta_k$

where

$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \sigma\}$

$0 < \alpha < \min\{\lambda_{\min}(-\Lambda), \max\{\lambda_{\max}(P), \lambda_{\max}(Q)\}\}$

so it has been proved that

$$\Delta V_k < -\alpha \eta_k^T \eta_k < -\frac{\alpha}{\sigma} V_k := -\psi V_k \tag{A.31}$$

Therefore by the definition (3.18) , it can be verified from the theorem (1.3.1) that the closed-loop nonlinear networked system (A.18) is exponentially meam square stable. This will end the proof. □

Next, the conditions have been found such that makes system (A.20) achieve H_∞ performance constraint (A.21). This has been discussed in the following thereom.

Theorem A.3.2. *Given communication channel parameters* $0 \ \leq \bar{\alpha} \leq 1 \ and \ 0 \ \leq \bar{\beta} \leq$

1,*and a scalar* $\gamma > 0$. *The uncertain closed loop netwroked nonlinear system (A.20) is uniformly stable and the* H_∞ *performance constraint (A.21) is achieved for all nonzero* w_k, *if there exists positive definite matrices* $P > 0,\ Q > 0$ *,real matrices*K, *and* L*; and positive real scalar* $\tau_1 > 0,\ \epsilon > 0$ *satisfying the matrix inequality shown in (A.32a)*

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} < 0 \tag{A.32a}$$

where

$$\Psi_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \tau_1^{0.5} G & * & * \\ 0 & PA - \bar{\beta}PBK & \bar{\beta}PBK & PD & 0 & -P & * \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 & -P \end{bmatrix},$$

$$\Psi_{22} = \begin{bmatrix} -Q & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \Delta_k & * & * & * & * & * & * & * \\ \beta_1 Q M_c^T & 0 & 0 & 0 & 0 & \Delta_k & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & \Delta_k & * & * & * & * & * \\ 0 & 0 & Q M_c^T & 0 & 0 & 0 & 0 & \Delta_k & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta_k & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta_k & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta_k & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Delta_k \end{bmatrix},$$

$$\Psi_{21} = \begin{bmatrix} 0 & \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & QA - \bar{\alpha} QLC_2 & Q(D - LD_2) & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \epsilon N_2 K & \epsilon N_2 K & 0 & 0 & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$\Psi_{12} = \Psi_{21}^T$ *and*

$$\alpha_1 = [(1-\bar{\alpha})\bar{\alpha}]^{1/2}\, and\, \beta_1 = \left[(1-\bar{\beta})\bar{\beta}\right]^{1/2}.$$

Proof. for any nonzero w_k, it follows from (A.30) that

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = \mathbb{E}\left\{[u_1]^T P\,[u_1] + [u_2]^T Q\,[u_2]\right\}$$

$$-{x_k}^T P x_k - e_k^T Q e_k^T + [u_3]^T [u_3] - \gamma^2 w_k^T w_k^T \triangleq \xi_k^T \Omega \xi_k$$

where

$$u_1 = ((A+\Delta A) - \bar{\beta}(B+\Delta B)K)f(x_k) + (\beta_k - \bar{\beta})(B+\Delta B)Ke_k$$

$$-(\beta_k - \bar{\beta})(B+\Delta B)Kf(x_k) + \bar{\beta}(B+\Delta B)Ke_k + Dw_k$$

$$u_2 = -(\beta_k - \bar{\beta})(B+\Delta B)Kf(x_k) - (\alpha_k - \bar{\alpha})LC_2 f(x_k)$$

$$+((A+\Delta A) - \bar{\alpha}LC_2)e_k + (\beta_k - \bar{\beta})(B+\Delta B)Ke_k$$

$$+(D - LD_2)w_k$$

$$u_3 = (C_1 - \Delta C_1)f(x_k) + D_1 w_k$$

For the simpilicity in deriving the equations, the following varaible has been put.

$$T_1 = ((A+\Delta A) - \bar{\beta}(B+\Delta B)K)f(x_k)$$

$$+\bar{\beta}(B+\Delta B)Ke_k + Dw_k$$

$$T_2 = (B+\Delta B)Ke_k - (B+\Delta B)Kf(x_k)$$

$$T_3 = ((A+\Delta A) - \bar{\alpha}LC_2)e_k + F_k + (D - LD_2)w_k$$

$$T_4 = LC_2 f(x_k)$$

So $\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\}$ can be rewritten as the following

$$
\begin{aligned}
&\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} \\
&\quad = \mathbb{E}\left\{\left[T_1 + (\beta_k - \bar{\beta})T_2\right]^T \times \left[PT_1 + (\beta_k - \bar{\beta})PT_2\right]\right\}\} \\
&\quad +\mathbb{E}\left\{\left[T_3 - (\alpha_k - \bar{\alpha})T_4 + (\beta_k - \bar{\beta})T_2\right]^T\right\} \\
&\quad \times\mathbb{E}\left\{\left[QT_3 - (\alpha_k - \bar{\alpha})QT_4 + (\beta_k - \bar{\beta})QT_2\right]\right\} \\
&\quad -{x_k}^T P x_k - e_k^T Q e_k^T \\
&\quad +[(C_1 - \Delta C_1) f(x_k) + D_1 w_k]^T [(C_1 - \Delta C_1) f(x_k) + D_1 w_k] \\
&\quad -\gamma^2 w_k^T w_k^T \\
&= \mathbb{E}\left\{T_1^T P T_1 + (\beta_k - \bar{\beta}) T_2^T P T_1\right\} + \mathbb{E}\left\{(\beta_k - \bar{\beta}) T_1^T P T_2 + (\beta_k - \bar{\beta})^2 T_2^T P T_2\right\} \\
&\quad +\mathbb{E}\left\{T_3^T Q T_3 - (\alpha_k - \bar{\alpha}) T_4^T Q T_3 + (\beta_k - \bar{\beta}) T_2^T Q T_3\right\} \\
&\quad -\mathbb{E}\left\{(\alpha_k - \bar{\alpha}) T_3^T Q T_4 - (\alpha_k - \bar{\alpha})^2 T_4^T Q T_4\right\} \\
&\quad -\mathbb{E}\left\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta}) T_2^T Q T_4\right\} \\
&\quad +\mathbb{E}\left\{(\beta_k - \bar{\beta}) T_3^T Q T_2 - (\beta_k - \bar{\beta})(\alpha_k - \bar{\alpha}) T_4^T Q T_2\right\} \\
&\quad +\mathbb{E}\left\{(\beta_k - \bar{\beta})^2 T_2^T Q T_2\right\} \\
&\quad -{x_k}^T P x_k \\
&\quad -e_k^T Q e_k^T \\
&\quad +[(C_1 - \Delta C_1) f(x_k) + D_1 w_k]^T [(C_1 - \Delta C_1) f(x_k) + D_1 w_k]
\end{aligned}
$$

$-\gamma^2 w_k^T w_k^T$

Where $\mathbb{E}\{(\beta_k - \bar{\beta})\} = \mathbb{E}(\beta_k) - \bar{\beta} = \bar{\beta} - \bar{\beta} = 0$, and the same thing for $\mathbb{E}\{(\alpha_k - \bar{\alpha})\} = 0$.

Then the following shape has been extracted.

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\}$$

$$= \left[T_1^T P T_1\right] + \left[\mathbb{E}\{(\beta_k - \bar{\beta})^2\} T_2^T P T_2\right] + \left[T_3^T Q T_3\right]$$

$$- \left[-\mathbb{E}\{(\alpha_k - \bar{\alpha})^2\} T_4^T Q T_4 + \mathbb{E}\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta})\} T_2^T Q T_4\right]$$

$$+ \left[-\mathbb{E}\{(\beta_k - \bar{\beta})(\alpha_k - \bar{\alpha})\} T_4^T Q T_2 + \mathbb{E}\{(\beta_k - \bar{\beta})^2\} T_2^T Q T_2\right]$$

$$-{x_k}^T P x_k - e_k^T Q e_k^T$$

$$+[(C_1 - \Delta C_1) f x_k + D_1 w_k]^T [(C_1 - \Delta C_1) f(x_k) + D_1 w_k] - \gamma^2 w_k^T w_k^T$$

From (4.9) and (4.14) $\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\}$ can be written as

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\}$$

$$= \left[T_1^T P T_1\right] + \left[\beta_1^2 T_2^T P T_2\right] + \left[T_3^T Q T_3\right]$$

$$- \left[-\alpha_1^2 T_4^T Q T_4 + \mathbb{E}\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta})\} T_2^T Q T_4\right]$$

$$+ \left[-\mathbb{E}\{(\beta_k - \bar{\beta})(\alpha_k - \bar{\alpha})\} T_4^T Q T_2 + \beta_1^2 T_2^T Q T_2\right]$$

$$-{x_k}^T P x_k - e_k^T Q e_k^T$$

$$+[(C_1 - \Delta C_1) f(x_k) + D_1 w_k]^T [(C_1 - \Delta C_1) f(x_k) + D_1 w_k] - \gamma^2 w_k^T w_k^T$$

Also, $\mathbb{E}\{(\alpha_k - \bar{\alpha})(\beta_k - \bar{\beta})\} = \mathbb{E}\{(\alpha_k \beta_k - \bar{\alpha}\beta_k - \alpha_k \bar{\beta} + \bar{\alpha}\bar{\beta})\} = \mathbb{E}\{(\alpha_k \beta_k)\} - \mathbb{E}\{\bar{\alpha}\beta_k\} -$

$\mathbb{E}\{\alpha_k\bar{\beta}\} + \mathbb{E}\{(\bar{\alpha}\bar{\beta})\}$

which lead to $\mathbb{E}\{(\alpha_k\beta_k)\} - \mathbb{E}\{\bar{\alpha}\beta_k\} - \mathbb{E}\{\alpha_k\bar{\beta}\} + \mathbb{E}\{(\bar{\alpha}\bar{\beta})\} = \bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta} - \bar{\alpha}\bar{\beta} + \bar{\alpha}\bar{\beta} = 0.$

This is done beacuse the property of the Bernoulli propabillity and the independence property of them

So

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2\mathbb{E}\{w_k^T w_k\}$$

$$= \left[T_1^T P T_1\right] + \left[\beta_1^2 T_2^T P T_2\right] + \left[T_3^T Q T_3\right]$$

$$+\alpha_1^2 T_4^T Q T_4 + \left[\beta_1^2 T_2^T Q T_2\right]$$

$$-{x_k}^T P x_k - e_k^T Q e_k^T$$

$$+[(C_1 - \Delta C_1)f(x_k) + D_1 w_k]^T[(C_1 - \Delta C_1)f(x_k) + D_1 w_k]$$

$$-\gamma^2 w_k^T w_k^T$$

Then, substitutions for the following varaibles have been made.

$$T_1 = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k) + \bar{\beta}(B + \Delta B)Ke_k + Dw_k$$

$$T_2 = (B + \Delta B)Ke_k - (B + \Delta B)Kf(x_k)$$

$$T_3 = ((A + \Delta A) - \bar{\alpha}LC_2)e_k + (D - LD_2)w_k$$

$$T_4 = LC_2 f(x_k)$$

This has given the following form.

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\}$$

$$= \left(((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k)\right)^T$$

$$+ \left(\bar{\beta}(B + \Delta B)Ke_k + Dw_k\right)^T$$

$$\times P \left(((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k)\right)$$

$$+P \left(\bar{\beta}(B + \Delta B)Ke_k + Dw_k\right)$$

$$+\beta_1^2 \left((B + \Delta B)Ke_k - (B + \Delta B)Kf(x_k)\right)^T$$

$$\times P \left((B + \Delta B)Ke_k - (B + \Delta B)Kf(x_k)\right)$$

$$+ \left(((A + \Delta A) - \bar{\alpha}LC_2)e_k\right)^T$$

$$\times Q \left(((A + \Delta A) - \bar{\alpha}LC_2)e_k + (D - LD_2)w_k\right)$$

$$+\alpha_1^2 \left(LC_2 f(x_k)\right)^T Q \left(LC_2 f(x_k)\right)$$

$$+\beta_1^2 \left((B + \Delta B)Ke_k - (B + \Delta B)Kf(x_k)\right)^T$$

$$\times Q \left((B + \Delta B)Ke_k - (B + \Delta B)Kf(x_k)\right)$$

$$-{x_k}^T P x_k - e_k^T Q e_k^T$$

$$+[(C_1 - \Delta C_1)f(x_k) + D_1 w_k]^T [(C_1 - \Delta C_1)f(x_k) + D_1 w_k]$$

$$-\gamma^2 w_k^T w_k^T$$

$$= (((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k))^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k)$$

$$+\bar{\beta}((B + \Delta B)Ke_k)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)f(x_k)$$

$$+(Dw_k)^T P((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k)$$

$$+(((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k))^T\bar{\beta}P(B+\Delta B)Ke_k$$

$$+\bar{\beta}((B+\Delta B)Ke_k)^T\bar{\beta}P(B+\Delta B)Ke_k$$

$$+(Dw_k)^T\bar{\beta}P(B+\Delta B)Ke_k$$

$$+(((A+\Delta A)-\bar{\beta}(B+\Delta B)K)f(x_k))^T PDw_k$$

$$+\bar{\beta}((B+\Delta B)Ke_k)^T PDw_k+(Dw_k)^T PDw_k$$

$$+\beta_1^2\left((B+\Delta B)Ke_k)^T P(B+\Delta B)Ke_k\right)$$

$$-\left(\beta_1^2((B+\Delta B)Kf(x_k)\right)^T P(B+\Delta B)Ke_k$$

$$-\beta_1^2(B+\Delta B)Ke_k)^T P\left((B+\Delta B)Kf(x_k)\right)^T$$

$$+\beta_1^2\left((B+\Delta B)Kf(x_k)\right)^T P(B+\Delta B)Kf(x_k)$$

$$+(((A+\Delta A)-\bar{\alpha}LC_2)e_k)^T Q((A+\Delta A)-\bar{\alpha}LC_2)e_k$$

$$+((D-LD_2)w_k)^T Q((A+\Delta A)-\bar{\alpha}LC_2)e_k$$

$$+(((A+\Delta A)-\bar{\alpha}LC_2)e_k)^T Q(D-LD_2)w_k$$

$$+((D-LD_2)w_k)^T Q(D-LD_2)w_k$$

$$+\alpha_1^2\left(LC_2f(x_k)\right)^T\left(QLC_2f(x_k)\right)$$

$$+\beta_1^2(B+\Delta B)Ke_k)^T Q(B+\Delta B)Ke_k$$

$$-\beta_1^2((B+\Delta B)Kf(x_k)^T Q(B+\Delta B)Ke_k$$

$$-\beta_1^2(B+\Delta B)Ke_k)^TQ(B+\Delta B)Kf(x_k)$$

$$+\beta_1^2(B+\Delta B)Kf(x_k)^TQ(B+\Delta B)Kf(x_k)$$

$$-{x_k}^TPx_k-e_k^TQe_k^T$$

$$+((C_1-\Delta C_1)f(x_k))^T(C_1-\Delta C_1)f(x_k)$$

$$+((C_1-\Delta C_1)f(x_k))^TD_1w_k$$

$$+(D_1w_k)^T(C_1-\Delta C_1)f(x_k)+(D_1w_k)^TD_1w_k$$

$$-\gamma^2w_k^Tw_k^T$$

This has resulted in the following matrix

$$\begin{aligned}
&\mathbb{E}\{V_{k+1}\}-\mathbb{E}\{V_k\}+\mathbb{E}\{Z_k^TZ_k\}-\gamma^2\mathbb{E}\{w_k^Tw_k\}= \\
&\begin{bmatrix} x_k \\ f(x_k) \\ e_k \\ w_k \end{bmatrix}^T \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & \Omega_{1\ 1} & * & * \\ 0 & \Omega_{2\ 1} & \Omega_{2\ 2} & * \\ 0 & \Omega_{3\ 1} & \Omega_{3\ 2} & \Omega_{3\ 3} \end{bmatrix} \begin{bmatrix} x_k \\ f(x_k) \\ e_k \\ w_k \end{bmatrix} \\
&\triangleq \zeta_k^T\Omega\zeta_k
\end{aligned} \tag{A.33}$$

where

$$\Omega_{1\ 1}=((A+\Delta A)-\bar{\beta}(B+\Delta B)K)^TP((A+\Delta A)-\bar{\beta}(B+\Delta B)K)$$

$$+\beta_1^2K^T(B+\Delta B)^TP(B+\Delta B)K$$

$$+\beta_1^2K^T(B+\Delta B)^TQ(B+\Delta B)K+\alpha_1^2C_2^TL^TQLC_2$$

$$+(C_1 - \Delta C_1)^T (C_1 - \Delta C_1)$$

$$\Omega_{2\,2} = \bar{\beta}^2 K^T (B + \Delta B)^T P (B + \Delta B) K$$

$$+\beta_1^2 K^T (B + \Delta B)^T P (B + \Delta B) K$$

$$+\beta_1^2 K^T (B + \Delta B)^T Q (B + \Delta B) K$$

$$+((A + \Delta A) - \bar{\alpha} L C_2)^T Q ((A + \Delta A) - \bar{\alpha} L C_2)$$

$$-Q$$

$$\Omega_{2\,1} = \bar{\beta} K^T (B + \Delta B)^T P ((A + \Delta A) - \bar{\beta} (B + \Delta B) K)$$

$$-\beta_1^2 K^T (B + \Delta B)^T P (B + \Delta B) K$$

$$-\beta_1^2 K^T (B + \Delta B)^T Q (B + \Delta B) K$$

$$\Omega_{3\,1} = D^T P ((A + \Delta A) - \bar{\beta} (B + \Delta B) K) + D_1^T (C_1 - \Delta C_1)$$

$$\Omega_{3\,2} = \bar{\beta} D^T P (B + \Delta B) K + (D - L D_2)^T Q ((A + \Delta A) - \bar{\alpha} L C_2)$$

$$\Omega_{3\,3} = D^T P D + (D - L D_2)^T Q (D - L D_2) + D_1^T D_1 - \gamma^2 I$$

In the same manner, It follows from (A.2) that

$$f(x_k)^T f(x_k) \leqslant x_k^T G^T G x_k \tag{A.34}$$

. This has resluted in

$$\zeta_k^T \begin{bmatrix} -G^TG & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \zeta_k \triangleq \zeta_k^T \Lambda_1^T \zeta_k \leq 0 \tag{A.35}$$

where $\zeta_k = \begin{bmatrix} x_k \\ f(x_k) \\ e_k \\ w_k \end{bmatrix}$ *By the S-procedure* $\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = \zeta_k^T \Omega \zeta_k < 0$ *with the constrains (A.35) holds if there exists positive-definite matrices*P, Q *and nonnegative scalars* $\tau_1 > 0$, *such that*

$$\Omega - \tau_1 \Omega_1 < 0 \tag{A.36}$$

$$\Omega - \tau_1\Omega_1 = \begin{bmatrix} -P + \tau_1 G^TG & 0 & 0 & 0 \\ 0 & \tilde{\Omega} & * & * \\ 0 & \Omega_2 & \breve{\Omega} & * \\ 0 & \Omega_4 & \Omega_3 & \hat{\Omega} \end{bmatrix} < 0$$

where

$$\tilde{\Omega} = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)$$

$$+\beta_1^2 K^T (B + \Delta B)^T P (B + \Delta B) K$$

$$+\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K + \alpha_1^2 C_2^T L^T Q L C_2$$

$$+(C_1-\Delta C_1)^T (C_1-\Delta C_1) + \tau_1 I$$

$$\breve{\Omega} = \bar{\beta}^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$+\beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$+\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$+((A+\Delta A)-\bar{\alpha} L C_2)^T Q ((A+\Delta A)-\bar{\alpha} L C_2) - Q$$

$$\hat{\Omega} = D^T P D + (D - L D_2)^T Q (D - L D_2) + D^T D_1 - \gamma^2 I$$

$$\Omega_2 = \bar{\beta} K^T (B+\Delta B)^T P ((A+\Delta A) - \bar{\beta}(B+\Delta B) K)$$

$$-\beta_1^2 K^T (B+\Delta B)^T P (B+\Delta B) K$$

$$-\beta_1^2 K^T (B+\Delta B)^T Q (B+\Delta B) K$$

$$\Omega_3 = \bar{\beta} D^T P (B+\Delta B) K$$

$$+(D - L D_2)^T Q ((A+\Delta A) - \bar{\alpha} L C_2)$$

$$\Omega_4 = D^T P ((A+\Delta A) - \bar{\beta}(B+\Delta B) K) + D_1^T (C_1 - \Delta C_1)$$

Or it can be in the equivalent form of a MI as shown below

$$\Omega - \tau_1 \Omega_1$$

$$= \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & \tau_1 I & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$+ \begin{bmatrix} \tau_1 G^T G^T & 0 & 0 & 0 \\ 0 & \tilde{\Omega} & * & * \\ 0 & \Omega_2 & \breve{\Omega} & * \\ 0 & \Omega_3 & \Omega_4 & \hat{\Omega} \end{bmatrix} < 0$$

where

$$\tilde{\Omega} = ((A + \Delta A) - \bar{\beta}(B + \Delta B)K)^T P((A + \Delta A)$$

$$-\bar{\beta}(B + \Delta B)K) + \beta_1^2 K^T (B + \Delta B)^T P(B + \Delta B)K$$

$$+\beta_1^2 K^T (B + \Delta B)^T Q(B + \Delta B)K$$

$$+\alpha_1^2 C_2^T L^T QLC_2 + C_1^T (C_1 - \Delta C_1)$$

$$\breve{\Omega} = \bar{\beta}^2 K^T (B + \Delta B)^T P(B + \Delta B)K$$

$$+\beta_1^2 K^T (B + \Delta B)^T P(B + \Delta B)K$$

$$+\beta_1^2 K^T (B + \Delta B)^T Q(B + \Delta B)K$$

$$+((A + \Delta A) - \bar{\alpha} LC_2)^T Q((A + \Delta A) - \bar{\alpha} LC_2)$$

$$\hat{\Omega} = D^T PD + (D - LD_2)^T Q(D - LD_2) + D^T D_1$$

$$\Omega_2 = \bar{\beta} K^T (B + \Delta B)^T P((A + \Delta A) - \bar{\beta}(B + \Delta B)K)$$

$$-\beta_1^2 K^T (B + \Delta B)^T P(B + \Delta B)K$$

$$-\beta_1^2 K^T(B+\Delta B)^T Q(B+\Delta B)K$$

$$\Omega_3 = D^T P((A+\Delta A) - \bar{\beta}(B+\Delta B)K) + D_1^T(C_1 - \Delta C_1)$$

$$\Omega_4 = \bar{\beta} D^T P(B+\Delta B)K$$

$$+(D - LD_2)^T Q((A+\Delta A) - \bar{\alpha} L C_2)$$

Also, It can be rewritten in the following form after applying the s-procedure.

$$\Omega - \tau_1 \Omega_1 = \begin{bmatrix} \Gamma_{11} & \Gamma_{12}^T \\ \Gamma_{12} & \Gamma_{22} \end{bmatrix} + \begin{bmatrix} 0 & \tilde{\Gamma}^T \\ \tilde{\Gamma} & 0 \end{bmatrix}$$

where

$$\Gamma_{11} = \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & \tau_1 I & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\Gamma_{22} = \begin{bmatrix} \tau_1^{0.5} G & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$\Gamma_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & PA - \bar{\beta}PBK & \bar{\beta}PBK & PD \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 \\ 0 & \beta_1 QBK & -\beta_1 Q(K & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 \\ 0 & 0 & QA - \bar{\alpha}QLC_2 & Q(D - LD_2) \\ 0 & C_1 & 0 & D_1 \end{bmatrix}$$

$$\tilde{\Gamma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & P\Delta A - \bar{\beta}P\Delta BK & \bar{\beta}P(\Delta B)K \\ 0 & \beta_1 P(\Delta B)K & -\beta_1 P(\Delta B)K \\ 0 & \beta_1 Q(\Delta B)K & -\beta_1 Q(\Delta B)K \\ 0 & 0 & 0 \\ 0 & 0 & Q(\Delta A) \\ 0 & (\Delta C_1) & 0 \end{bmatrix}$$

Based on (A.6) and Lemma (A.22) with Lemma (A.23) MI can be rewritten in the following form:

$$\Omega - \tau_1\Omega_1 = \begin{bmatrix} \Gamma_{11} & \Sigma^T \\ \Sigma & \Gamma_{22} \end{bmatrix} + \left[\tilde{\Theta}\right] \Delta \left[\breve{\Theta}\right] + \left[\breve{\Theta}\right]^T \Delta^T \left[\tilde{\Theta}\right]^T$$

where

$$\Gamma_{11} = \begin{bmatrix} -P & 0 & 0 & 0 \\ 0 & \tau_1 I & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Gamma_{22}=\begin{bmatrix} \tau_1^{0.5}G & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -P & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Q & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$\Sigma=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & PA-\bar{\beta}PBK & \bar{\beta}PBK & PD \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 \\ 0 & \beta_1 QBK & -\beta_1 QBK & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 \\ 0 & 0 & QA-\bar{\alpha}QLC_2 & Q(D-LD_2) \\ 0 & C_1 & 0 & D_1 \end{bmatrix}$$

$$\tilde{\Theta}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (PM_c) & -\bar{\beta}\,(PM_c) & 0 & 0 \\ 0 & \beta_1\,(PM_c) & 0 & 0 \\ 0 & \beta_1\,(QM_c) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (QM_c) \\ 0 & 0 & N_3 & 0 \end{bmatrix},$$

$$\breve{\Theta} = \begin{bmatrix} (N_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (N_2K) & (N_2K) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ M_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Delta = \begin{bmatrix} \Delta_k & 0 & 0 & 0 \\ 0 & \Delta_k & 0 & 0 \\ 0 & 0 & \Delta_k & 0 \\ 0 & 0 & 0 & \Delta_k \end{bmatrix}$$

While from (A.7) $(\Delta_k \Delta_k^T) < I$, *it can be held for the condition that there is* $\epsilon > 0$ *such that*

$$\Omega - \tau_1 \Omega_1 = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} < 0,$$

where

$$\Psi_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \tau_1^{0.5} G & * & * \\ 0 & PA - \bar{\beta} PBK & \bar{\beta} PBK & PD & 0 & -P & * \\ 0 & \beta_1 PBK & -\beta_1 PBK & 0 & 0 & 0 & -P \end{bmatrix},$$

$$\Psi_{22} = \begin{bmatrix} -Q & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -Q & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * & * \\ \beta_1 Q M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & Q M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Psi_{21} = \begin{bmatrix} 0 & \beta_1 QBK & -\beta_1 QBK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 QLC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & QA - \bar{\alpha} QLC_2 & Q(D - LD_2) & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \epsilon N_2 K & \epsilon N_2 K & 0 & 0 & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Psi_{12} = \begin{bmatrix} * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * & * & * & * \\ 0 & * & * & * & * & * & * & * & * & * & * & * \end{bmatrix}.$$

This will give the same MI shown in (A.32a).

Then, the congruence transformation has been made with the matrix

$daig\{I, I, I, I, , I, P^{-1}, P^{-1}\ , Q^{-1}, Q^{-1}, Q^{-1}, I, I, I, I, I, I, I, I, I\}$. *The resulted MI will be*

$\Omega - \tau_1\Omega_1 =$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} < 0 \qquad \text{(A.37)}$$

where

$$\Psi_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \tau_1^{0.5} G & * & * \\ 0 & A - \bar{\beta}BK & \bar{\beta}BK & D & 0 & -P^{-1} & * \\ 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & -P^{-1} \end{bmatrix},$$

$$\Psi_{22} = \begin{bmatrix} -Q^{-1} & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -Q^{-1} & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -Q^{-1} & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * & * \\ \beta_1 M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix}$$

$$\Psi_{21} = \begin{bmatrix} 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A - \bar{\alpha} LC_2 & D - LD_2 & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \epsilon N_2 K & \epsilon N_2 K & 0 & 0 & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \textit{ and}$$

$$\Psi_{12} = \Psi_{21}^T$$

So by using schur complement, the inequality in (A.32a) is equivalent to (A.37) thus we have

$$\zeta_k^T \Omega \zeta_k^T < 0 \tag{A.38}$$

so we can conclude from (A.33) and (A.38) that

$$\mathbb{E}\{V_{k+1}\} - \mathbb{E}\{V_k\} + \mathbb{E}\{Z_k^T Z_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} < 0 \tag{A.39}$$

Now, summing (A.39) from 0 to ∞ *with respect to* k *yield*

$$\sum_{k=0}^{\infty} \mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{w_k^T w_k\} - \mathbb{E}\{V_0\} - \mathbb{E}\{V_\infty\} \tag{A.40}$$

since the closed-loop networke nonlinear system (A.20) is exponentially mean square stable and $\eta_0 = 0$ *it is straight forword to conclude that*

$$\sum_{k=0}^{\infty} \mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{w_k^T w_k\} \tag{A.41}$$

Which means the specified H_∞ *performance constraints (A.21) is achieved. This ends the proof.* □

In order to solve this MI and to design the controller easily, it is needed to derive the explicit expression of the controller parameter in the terms of the LMI slover, such as Matlab Toolbox

In order to find LMI and to design the controller without conservation,it can be modified by the following two approaches

1- Removing the ϵ either by suggesting $\epsilon = 1$.or making $\bar{K} = \epsilon K$

2-Changing P^{-1}, Q^{-1} to an equivalent matrix .

3- Replacing $\tau_1^{0.5}$ with $\bar{\tau} = \tau_1^{0.5}$.

The first approach for P^{-1}, Q^{-1}can be made from the matrix (A.37), which can be held if there is $\Omega_3 > 0$ such that

$\hat{\Omega} - \Omega_3 < 0$

where $\Omega_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \tilde{N} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\tilde{N} = \begin{bmatrix} N_{11} & 0 & 0 & 0 & 0 \\ 0 & N_{11} & 0 & 0 & 0 \\ 0 & 0 & N_{22} & 0 & 0 \\ 0 & 0 & 0 & N_{22} & 0 \\ 0 & 0 & 0 & 0 & N_{22} \end{bmatrix},$$

0 is zero matrix with proper dimensions and $\hat{\Omega}$ is the Schur complement of $\Omega - \tau_1\Omega_1 -$

$\tau_2\Omega_2$

N_{11}, N_{22} are matrices with the same dimensions of P, Q respectively such that

$N_{11} = (\tilde{N}_1 - P^{-1})$

$N_{22} = (\tilde{N}_2 - Q^{-1})$

where $\tilde{N}_1$and $\tilde{N}_2$ are positive-definite matrices. After applying these matrix , the first approach with the suggestion $\epsilon = 1$ has been in the following approach

$$\begin{bmatrix} \Xi_{11} & \Upsilon_2 \\ \Upsilon_1 & \Xi_{22} \end{bmatrix} < 0 \tag{A.42}$$

where

,

$$\Xi_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \bar{\tau}G & * & * \\ 0 & A - \bar{\beta}BK & \bar{\beta}BK & D & 0 & -\tilde{N}_1 & * \\ 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & -\tilde{N}_1 \end{bmatrix}$$

$$\Xi_{22} = \begin{bmatrix} -\tilde{N}_2 & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -\tilde{N}_2 & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\tilde{N}_2 & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * & * \\ \beta_1 M_c^T & 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & -I & * & * & * & * & * \\ 0 & 0 & M_c^T & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A - \bar{\alpha} LC_2 & D - LD_2 & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_2 K & N_2 K & 0 & 0 & 0 & 0 & 0 \\ M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$\Upsilon_2 = \Upsilon_1^T.$$

Also, the approach can be rewritten with considering $\bar{K} = \epsilon K$ and the LMI as the following.

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Upsilon_1^T \\ \Upsilon_1 & \tilde{\Xi}_{22} \end{bmatrix} < 0 \tag{A.43}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \bar{\tau} G & * & * \\ 0 & A - \bar{\beta} BK & \bar{\beta} BK & D & 0 & -\tilde{N}_1 & * \\ 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & -\tilde{N}_1 \end{bmatrix},$$

$$\tilde{\Xi}_{22} = \begin{bmatrix} -\tilde{N}_2 & * & * & * & * & * & * & * & * & * & * & * \\ 0 & -\tilde{N}_2 & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & -\tilde{N}_2 & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * & * \\ \beta_1 M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix}$$

,

$$\Upsilon_1 = \begin{bmatrix} 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A - \bar{\alpha} LC_2 & D - LD_2 & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_2\bar{K} & N_2\bar{K} & 0 & 0 & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This format is needed to derive the explicit expression of the controller parameter in the terms of the LMI slover, such as Matlab Toolbox.It is noted in this format that there are unknown parameters matrices more than the symmetric matrices P and Q which are $\tilde{N}_1$and $\tilde{N}_2$. with $\bar{K} = \epsilon K$ as second form for it.

Even though it can be solved by the LMI solver but to reduce the number of the unknown paramete matrices the following LMI is suggested

In order to solve this LMI and to design the controller easily the following modification is suggested by [42].

while $-P^{-1}$ and $-Q^{-1}$are replaced by $2I - P$ and $2I - Q$

Therefore LMI can be rewritten in the following form with considering $\epsilon = 1$ as shown in the following

$$\begin{bmatrix} \tilde{\tilde{\Xi}}_{11} & \Upsilon_2 \\ \Upsilon_1 & \breve{\Xi}_{22} \end{bmatrix} < 0 \tag{A.44}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \bar{\tau} G & * & * \\ 0 & A - \bar{\beta} BK & \bar{\beta} BK & D & 0 & 2I - P & 0 \\ 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 2I - P \end{bmatrix},$$

$$\tilde{\Xi}_{22} = \begin{bmatrix} 2I-Q & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 2I-Q & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 2I-Q & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * & * \\ \beta_1 M_c^T & 0 & 0 & 0 & 0 & -I & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & -I & * & * & * & * & * \\ 0 & 0 & M_c^T & 0 & 0 & 0 & 0 & -I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A - \bar{\alpha} LC_2 & D - LD_2 & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_2 K & N_2 K & 0 & 0 & 0 & 0 & 0 \\ M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$\Upsilon_2 = \Upsilon_1^T$.

It can be in other form by suggestion $\bar{K} = \epsilon K$ which leads to the following LMI

$$\begin{bmatrix} \tilde{\Xi}_{11} & \Upsilon_3^T \\ \Upsilon_3 & \breve{\Xi}_{22} \end{bmatrix} < 0 \tag{A.45}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & \tau_1 I & * & * & * & * & * \\ 0 & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * & * \\ 0 & 0 & 0 & 0 & \bar{\tau} G & * & * \\ 0 & A-\bar{\beta}BK & \bar{\beta}BK & D & 0 & 2I-P & 0 \\ 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 2I-P \end{bmatrix},$$

$$\breve{\Xi}_{22} = \begin{bmatrix} 2I-Q & * & * & * & * & * & * & * & * & * & * & * \\ 0 & 2I-Q & * & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 2I-Q & * & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & I & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * & * \\ \beta_1 M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * & * & * \\ 0 & 0 & 0 & N_3^T & 0 & 0 & -\epsilon I & * & * & * & * & * \\ 0 & 0 & M_c^T & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} 0 & \beta_1 BK & -\beta_1 BK & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 LC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A-\bar{\alpha}LC_2 & D-LD_2 & 0 & 0 & 0 \\ 0 & C_1 & 0 & D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & PM_c^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -PM_c^T & \beta_1 PM_c^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ N_2\bar{K} & N_2\bar{K} & 0 & 0 & 0 & 0 & 0 \\ \epsilon M_c^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \epsilon N_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$\Upsilon_2 = \Upsilon_1^T$

As a results to that ,in order to establish the observer-based controller (A.12) and (A.13) for networked nonlinear system (A.1) under the H_∞ peformance constraint (A.21) with minum γ., it is supposed to consider the following optimization problem

$$\min_{P>0,Q>0\tilde{N}_1>0,\tilde{N}_2>0,\tau_1>0,\tau_2>0,} \gamma \tag{A.46}$$

subject to inequality given in (A.42),(A.43),(A.44)or(A.45)

Finally,it concludes with the following two theorems

Corollary A.3.1. *Given communication channel parametes* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$, *If the optimization problem (A.46) is feasible,the observer-based control law (A.13) with controller parameters* K *and* L *that can be found by LMI described in (A.42) or (A.43) will uniformly stabilize the uncertain netwroked nonlinear system (A.1) with minimum H*$_\infty$ *Performance bound* $\gamma_{\min}$.

Corollary A.3.2. *Given communication channel parametes* $0 \leq \bar{\alpha} \leq 1 and\ 0 \leq \bar{\beta} \leq 1$, *If the optimization problem (A.46) is feasible,the observer-based control law (A.13) with controller parameters* K *and* L *that can be found by LMI described in (A.44) or(A.45) will uniformly stabilize the uncertain netwroked nonlinear system (A.1) with minimum H*$_\infty$ *Performance bound* $\gamma_{\min}$.

A.4 Conclusion

At the end of this appendex it can be concluded that an observer-based H_∞ controller is designed for a class of uncertain NNCS. This class contains a nonlinear state varaible exposed to the uncertainty but it has had a constant matrix which can be taken as common factor.The packet loss change has been described as Bernoulli distribution. The probability of the packet loss from the sensor to the controller was different from the controller to the actuator. Certain conditions were suggested and it is needed to test their validity.There are suggestions to bring a numerical example suitable to its conditions in the future. It has been based on the work of[4],[5] and [51] to create four LMI approaches. These LMIs have been created to suggest the stability for this system with H_∞ minimization performance. What is left in this appendex is to chose a numerical example and to make a simulation to test the resuts.

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